Problème 1 : Syringe *(18 points)*

We would like to do some simple experiments with a syringe. You can find all the material for the experiment in the envelope that we sent to you. You will need the following material:

- 1 syringe
- 1 plastic block
- 4 sheets of graph paper
- 1 chewing gum

Note: You must ONLY use the 4 items listed above to make measurements for this experiment, so no pencil, no set-square or ruler, nothing else. Also none of the other items in the envelope. For the documentation of the tasks on paper you may of course use writing utensils, set-square, etc.

To solve the problems you get the following information:

- Inside radius of syringe : $r = 7.4$ mm
- If you need other quantities, make a reasonable assumption and indicate it as such.

Partie A. Friction (9 points)

i. (9 pts) Measure the static friction force of the piston in the syringe. Document your solution as follows:

1. Describe and do a sketch of your setup and your approach for the measurements.
2. Calculate the static friction force (the calculation method must be documented).
3. Do an error calculation and use it to justify your approach.

Solutions

Note: If the students use material for the experiment we did not provide, this problem gives zero points (the next problem hast to be considered separately)!

Beschreibe und skizziere den Messaufbau und Dein Vorgehen beim Messen

Seal the syringe with a finger and pull or push the piston a bit. Then the air outside has more or less pressure respectively than inside. The pressure difference is pushing the piston back. If the static friction is large enough the piston does not move. Pull/Push the piston a bit more and check again if the piston moves. Repeat this procedure till the pistons moves. At the maximal displacement of the piston where it does not move back, the static friction is as big as the force due to the pressure difference.

Note: if the piston is pulled or pushed so far that it glides back until it stands still, there is dynamic and not static friction involved!

Note: it is easier to seal the syringe while pulling the piston due to the suction. The tighter seal leads to more precise measurements. On the other hand, pushing the piston allows for a larger initial Volume leading to a smaller error (see error calculations). It is unclear which approach leads to the better overall results. Therefore both approaches are accepted as equal. The participants are not expected to do any reasoning about this trade-off.

The distribution of points is the following:

- Idea of sealing the syringe with the finger and pull/push the piston. **0.5**

- Effectively measuring the static friction (and not the dynamic one). **0.5**
- Read off the volume on the syringe. **0.5**
- Nice, clear and big sketch of the experimental setup. **0.5**

Bestimme die Haftreibung (Rechenweg muss ersichtlich sein)

- The force due to the pressure difference on the piston is $F_{\Delta p} = \Delta p A$ where Δp is the pressure difference and $A = \pi r^2$ the cross-section area of the syringe (computation of the area itself gives no points (r is the inner radius of the cylinder)). **0.5**
- Assuming isothermal expansion/compression $p_f V_f = p_o V_o = cte$, where V_o is the initial volume when sealing the syringe, p_o is the room pressure and V_f and p_f are the final volume and pressure in the syringe at maximal displacement. **0.5**
- The pressure difference is $\Delta p = p_f - p_o = p_o(V_f - V_o)/V_f$. **0.5**
- Choose suitable room pressure p_o and state that this is an assumption **0.5**
- Repeating the measurement enough often and taking the mean: Repeat 5 times if less often: each time less: minus 0.5 pt (only measuring once, 0 pt). **2**

9.6	9.4	9.4	9.7	9.6	9.7	9.5	9.6	9.4	9.5

Tab. 1: V_f in ml for $V_o = 9$ ml and pulling, Mean : 9.54 ml, STD : 0.12 ml

9.6	9.5	9.5	9.7	9.6	9.7	9.6	9.4	9.6	9.7

Tab. 2: V_f in ml for $V_o = 10$ ml and pushing, Mean : 9.58 ml, STD : 0.09 ml

Note: with the 10 ml syringe and a 0.2 ml scale on it, 0.1 ml is approximately the maximum precision that can be read.

- The friction force is $F_f = F_{\Delta p}$ (pulling) or $F_f = -F_{\Delta p}$ (pushing). **0.5**
- Numerical value $F_f = \pi r^2 p_o \frac{V_f - V_o}{V_f} = 0.97$ N (pulling) or $F_f = \pi r^2 p_o \frac{V_o - V_f}{V_f} = 0.75$ N (pushing). Assuming $p_o = 10^5$ pa.
  Note: also give this point if only measured once. **0.5**

Note: Possible explanations for the difference between the two friction forces might be: A) Measured at different point in syringe, B) The rubber joint is not symmetric, C) Asymmetry between inside-/outside-pressure.

Führe eine Fehlerrechnung durch und Begründe damit dein Vorgehen
There are different methods to estimate the error. All methods are considered equivalent for the estimation. **1**

- compute the standard deviation from the different measurements. For results see table 1 and 2.

- Using Gaussian error propagation

$$\Delta F = \sqrt{\left(\frac{A(V_f - V_0)}{V_f}\Delta p_o\right)^2 + \left(\frac{p_o(V_f - V_0)}{V_f}\Delta A\right)^2 + \left(\frac{p_o A}{V_f}\Delta V_0\right)^2 + \left(\frac{p_o A V_0}{V_f^2}\Delta V_f\right)^2}$$
$$\approx 0.28 \text{ N}$$

  assuming $p_0 = 10^5$ pa, $\Delta p_o = 0.13 \times 10^5$ pa (pressure variation between 400m and 1500m of altitude above sea-level, assuming the location of the experiment is unknown, additionally there are weather effects on the pressure, other reasonable choices for Δp_o are accepted as well), $\Delta A = 4.6$ mm^2 (based on $\Delta r = 0.1$ mm), $\Delta V_o = 0.1$ ml, $\Delta V_f = 0.1$ ml $V_o = 9$ ml and $V_f = 9.54$ ml.

- similar to Gaussian error propagation but without the square root but absolute values.

$$\Delta F = \left|\frac{A(V_f - V_o)}{V_f}\Delta p_o\right| + \left|\frac{p_o(V_f - V_o)}{V_f}\Delta A\right| + \left|\frac{p_o A}{V_f}\Delta V_o\right| + \left|\frac{p_o A V_o}{V_f^2}\Delta V_f\right|$$
$$\approx 0.50 \text{ N}$$

  same assumptions as above.

The argumentation for measuring at bigger volume is: **1**

- From the error calculation we see that the error can be minimised by performing this measurement at bigger volumes. The error on reading off the volume (ΔV_f and ΔV_0) are independent of the volume. So the last two terms in the error estimation gets smaller for big volumes (V_f and V_0).

- The first error calculation does not allow to justify the choice of the initial volume. An intuitive explanation is also accepted, if correct.

Note: If a student makes a mistake in the error calculation but the argumentation for the measurement is correct, these points shall be given (but not for the error calculation).

Partie B. Mass (9 points)

i. (9 pts) Measure the mass ratio of the piston and the cylinder of the syringe. Document your solution as follows:

1. Describe and do a sketch of your setup and your approach for the measurements.
2. Display your data points graphically and calculate the mass ratio (the method of your solution must be documented).

Solutions

Note: If the students use material for the experiment we did not provide, this problem gives zero points (the previous problem hast to be considered separately)!

Beschreibe und Skizziere den Messaufbau und Dein Vorgehen beim Messen

First we denote the different quantities involved, see figure 1. Note: There are different choices possible!

- m_p the mass of the piston
- m_c the mass of the cylinder
- r_p the distance between the front of the piston and the center of mass of the piston

- r_c the distance between the front of the cylinder and the center of mass of the cylinder
- x the distance between the front of the piston and the cylinder.
- y the distance between the center of mass of the whole syringe and the front of the cylinder

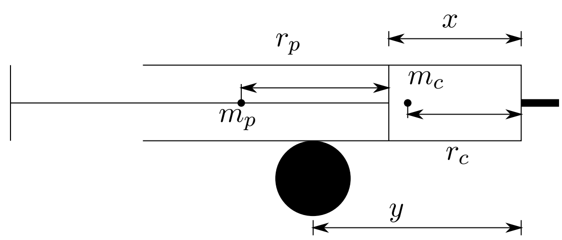

Fig. 1: Sketch of the setup in the second task. The small circles denote the center of mass of the piston and the cylinder respectively.

The idea is to find the position of the center of mass of the whole syringe y for different x. In the x-y plot, the slope is related to the searched mass ration m_p/m_c, details see below. The center of mass of the whole syringe can be found by balancing the syringe on the block. The distances x and y can be measured using the volume scale (for y one has to estimate where the syringe touches the tip of the block).
The distribution of points is the following:

- Introduce used variables clearly (the variable used might be defined slightly different) **0.5**
- Good drawing (big, clearly labelled) **0.5**
- Idea of using a balance **1**
- Use volume scale as length scale (using $x = V_x/A$ where V_x is the volume at x, analogous y) . **0.5**

Stelle Deine Datenpunkte grafisch dar und berechne das Masseverhältnis (Lösungsweg muss ersichtlich sein)
The position of the center of mass of the whole syringe can be computed using

$$y = \frac{m_p(r_p + x) + m_c r_c}{m_p + m_c} = \frac{m_p}{m_p + m_c}x + \frac{m_p r_p + m_c r_c}{m_p + m_c}.$$

The inverse of the slope s is therefore

$$\frac{\Delta x}{\Delta y} = \frac{1}{s} = 1 + \frac{m_c}{m_p}.$$

The distribution of points is the following:

- Formula for y (if different approach chosen to get $\frac{m_c}{m_p}$ distribute points accordingly) **1**
- Formula putting $\frac{m_c}{m_p}$ and $\frac{\Delta x}{\Delta y}$ into relation. **0.5**
- Enough data points: 5 data points: 1.5 pt, , for 4: 1 pt, for 3: .5, less than 3: zero (no regression line possible) **1.5**
- Plot (each 0.5 points: Axis labelled, big drawing, data points correct) **1.5**

x	0	1	2	3	4	5	6	7
y	5.9	6.6	7.1	7.6	8.0	8.5	9.0	9.6

Tab. 3: x and y in ml

For the calculation of the numerical value of $\frac{m_c}{m_p}$ there are two options:

- We guess a trend line in the x-y plot. From this trend line we compute the inverse slope and get the searched ratio by subtracting one.
  - Idea of plotting and read read of slope (give these points also if the student does not get the correct result) **1**
  - Drawing reasonable trend line. **0.5**
  - Getting the right result. **0.5**
- Computing the slope numerically using linear regression by hand or with the calculator.
  - Describing clearly their method (e.g. mention calculator is fine). **1**
  - Getting the right result. **1**
- Note: Students might also compute the slope between multiple pairs of data points, compute the mass ratio and take the average over all the values gained form the pairs. This is not valid. One simple example shows why. If due to measurement errors one Δx tends to 0 then the corresponding slope tends to infinity. No mean including this value will result in a reasonable result.
- Linear regression of the form $y = Bx + A$ based on the values from table 3 gives the results $A = 6.02$ ml and $B = 0.51$. Therefore $\frac{m_c}{m_p} = \frac{\Delta x}{\Delta y} - 1 = \frac{1}{B} - 1 = 0.98$

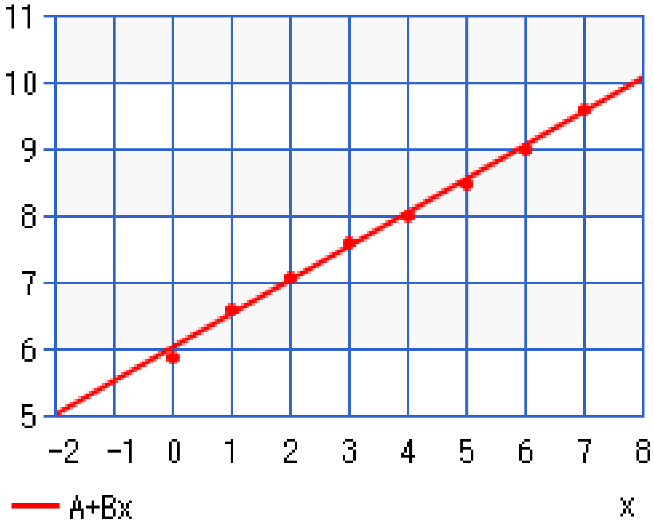

Fig. 2: Plotted values with least-squares linear trend-line.