Physics Olympiad

Second Round

18 January 2023

Part 1	**:**	**21 MC questions**
Duration	:	60 minutes
Total	:	21 points (21×1)
Part 2	**:**	**3 long problems**
Duration	:	120 minutes
Total	:	48 points (3×16)
Authorized material	:	Calculator without database
		Writing and drawing material

Good luck!

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Natural constants

Caesium hyperfine frequency	$\Delta\nu_{\mathrm{Cs}}$	9.192 631 770	$\times 10^{9}$	s^{-1}
Speed of light in vacuum	c	2.997 924 58	$\times 10^{8}$	$\mathrm{m} \cdot \mathrm{s}^{-1}$
Planck constant	h	6.626 070 15	$\times 10^{-34}$	$\mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-1}$
Elementary charge	e	1.602 176 634	$\times 10^{-19}$	$\mathrm{A} \cdot \mathrm{s}$
Boltzmann constant	k_{B}	1.380 649	$\times 10^{-23}$	$\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-2}$
Avogadro constant	N_{A}	6.022 140 76	$\times 10^{23}$	mol^{-1}
Luminous efficacy of radiation	K_{cd}	6.83	$\times 10^{2}$	$\mathrm{cd} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{3} \cdot \mathrm{sr}$
Magnetic constant	μ_0	1.256 637 062 12(19)	$\times 10^{-6}$	$\mathrm{A}^{-2} \cdot \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$
Electric constant	ε_0	8.854 187 812 8(13)	$\times 10^{-12}$	$\mathrm{A}^{2} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-3} \cdot \mathrm{s}^{4}$
Gas constant	R	8.314 462 618...		$\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{mol}^{-1} \cdot \mathrm{s}^{-2}$
Stefan-Boltzmann constant	σ	5.670 374 419...	$\times 10^{-8}$	$\mathrm{K}^{-4} \cdot \mathrm{kg} \cdot \mathrm{s}^{-3}$
Gravitational constant	G	6.674 30(15)	$\times 10^{-11}$	$\mathrm{kg}^{-1} \cdot \mathrm{m}^{3} \cdot \mathrm{s}^{-2}$
Electron mass	m_{e}	9.109 383 701 5(28)	$\times 10^{-31}$	kg
Neutron mass	m_{n}	1.674 927 498 04(95)	$\times 10^{-27}$	kg
Proton mass	m_{p}	1.672 621 923 69(51)	$\times 10^{-27}$	kg
Standard acceleration of gravity	g_{n}	9.806 65		$\mathrm{m} \cdot \mathrm{s}^{-2}$

Multiple Choice

Duration: 60 minutes
Marks: 21 points (1 point for each correct answer)

- Multiple-Choice **(MC)** questions have several statements, of which **exactly one** is correct. If you mark exactly the right answer on the answer sheet, you get one point, otherwise zero.

Question 1.1 (MC)

If the world population was distributed uniformly on the landmasses of the Earth, what area would be available for each person?

A) $1.8 \times 10^4\,\mathrm{m}^2$ B) $1.8 \times 10^6\,\mathrm{m}^2$

C) $1.8 \times 10^8\,\mathrm{m}^2$ D) $1.8 \times 10^{10}\,\mathrm{m}^2$

Question 1.2 (MC)

What is the difference in energy between a spin-up and spin-down electron?

A) $E = \frac{e}{m_e}\hbar^2 B$ B) $E = \frac{e}{m_e}\hbar B$

C) $E = \frac{e^2}{m_e}\hbar B$ D) $E = \frac{e^2}{m_e^2}\hbar B$

E) $E = \frac{e^2}{m_e}\hbar^2 B$

Question 1.3 (MC)

Let the vector $\vec{a} \neq \vec{0}$. Which equation is correct?

A) $(\vec{a} + \vec{a}) + \vec{a} = \vec{0}$ B) $(\vec{a} + \vec{a}) \times \vec{a} = \vec{0}$

C) $(\vec{a} + \vec{a}) \cdot \vec{a} = 0$ D) $(\vec{a} \times \vec{a}) + \vec{a} = \vec{0}$

E) $(\vec{a} \cdot \vec{a})\,\vec{a} = \vec{0}$

Question 1.4 (MC)

If the Earth were spinning faster around itself, the geostationary orbit would be...

A) lower. B) unchanged.

C) higher.

Question 1.5 (MC)

Asterix and Obelix decide to play pétanque with menhirs. Obelix (as usual very strong) throws the menhir with a speed of $100\,\mathrm{m \cdot s^{-1}}$ with an angle of 45° towards the horizon away from himself. The menhir has a mass of 100 kg. Asterix calculated the maximal height of the menhir during its flight. What value did he find?

A) 25 m B) 255 m

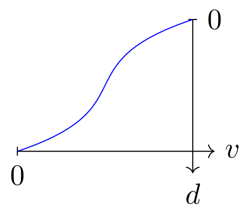
C) 500 m D) 980 m

E) The menhir reaches the Moon.

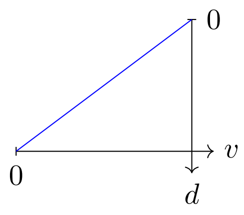
Question 1.6 (MC)

Cloé measures the speed v of the water in a river of infinite width at different depths d. What is the speed profile she most likely observes?

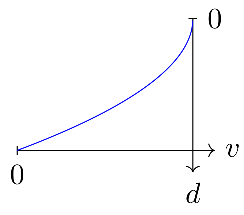
A)



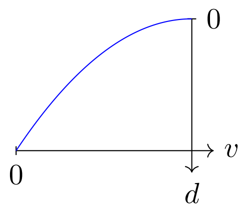
B)



C)



D)



Question 1.7 (MC)

A ball elastically strikes another ball of the same mass initially at rest. The final velocity of the second ball is half the initial velocity of the first ball. What is the angle between the final velocity of the second ball and the initial velocity of the first ball?

A) 0° B) 30° C) 45° D) 60° E) 90°

Question 1.8 (MC)

A wall is moving leftwards with a constant velocity u. A ball is moving rightwards towards the wall at a velocity v. What is the velocity of the ball after the elastic collision?

A) v leftwards

B) u leftwards

C) $v + u$ leftwards

D) $v + 2u$ leftwards

E) $v + \frac{u}{2}$ leftwards

Question 1.9 (MC)

Two point masses m and M are separated by R. M is fixed in place and m is initially at rest. m is then attracted towards M via the gravitational force. How much time is needed for m to hit M?

A) $\sqrt{\frac{\pi^2 R^3}{8GM}}$ B) $\sqrt{\frac{\pi^2 R^3}{2GM}}$ C) $\sqrt{\frac{\pi^2 R^3}{GM}}$

D) $\sqrt{\frac{2\pi^2 R^3}{GM}}$ E) $\sqrt{\frac{8\pi^2 R^3}{GM}}$

Question 1.10 (MC)

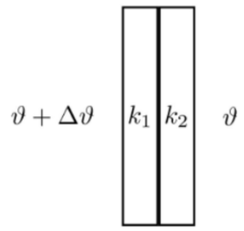
An ice calorimeter is an apparatus to measure the specific heat capacity of substances. In essence, the substance is heated up and then placed on a block of ice at a temperature of 0 °C. Then one measures the ammount of molten water. We perform this experiment with a sample of 100 g of lead. We heat the dample to a temperature of 50 °C and measure 1.93 g of molten water. What is the specific heat capacity of lead? The heat of fusion of water is $333.7\,\mathrm{J \cdot g^{-1}}$.

A) $0.129\,\mathrm{J \cdot g^{-1} \cdot K^{-1}}$ B) $0.775\,\mathrm{J \cdot g^{-1} \cdot K^{-1}}$

C) $1.29\,\mathrm{J \cdot g^{-1} \cdot K^{-1}}$ D) $7.75\,\mathrm{J \cdot g^{-1} \cdot K^{-1}}$

Question 1.11 (MC)

Let us consider two plates, each with a surface of $1\,\mathrm{m^2}$ and a thickness of 1 cm. These plates have a thermal conductivity of $k_1 = 0.7\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$ and $k_2 = 1.0\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$, respectively. When the plates are in contact, what is the heat flux through them if the temperature difference is $\Delta\vartheta = 10\,\mathrm{K}$?

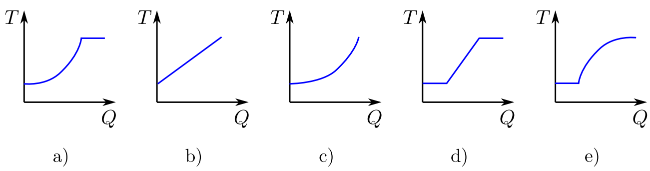


A) 0.21 W B) 4.1 W C) 0.41 kW

D) 1.7 kW E) 21 kW

Question 1.12 (MC)

A mixture of water and ice is heated. Which graph best describes the relationship between the temperature T of the system and the supplied heat Q?



A) a B) b C) c D) d E) e

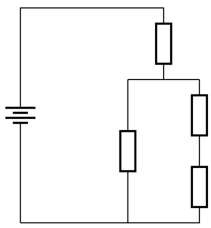
Question 1.13 (MC)

Alice has recently heard about the golden ratio φ. She remembers learning that it is possible to write φ as a continued fraction:

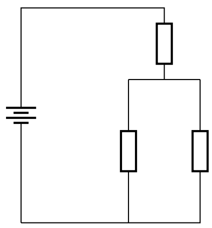
$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\cdots}}}}}.$$

She then realizes that she could use her resistors of exactly $R = 1\,\Omega$ to build a circuit with equivalent resistance approximating the golden ratio arbitrarily well. She does not have infinitely many resistors, though. Which of the following circuits should Alice build to get a total resistance closest to $\varphi\Omega$?

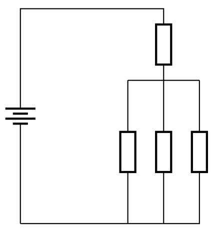
A)



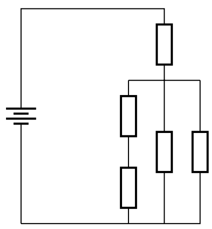
B)



C)

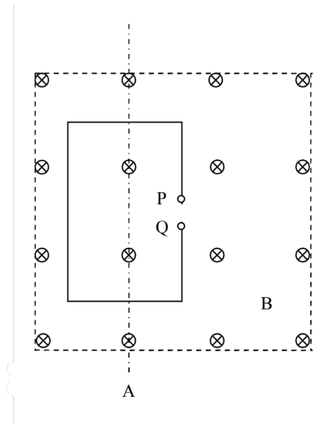


D)



Question 1.14 (MC)

A rectangular conductor loop is placed in a homogeneous magnetic field perpendicular to the drawing plane and pointing into the drawing plane (see sketch). The magnetic field B is generated by a coil through which a current I flows.



Which of the following statements is false? A voltage will be induced in the conductor loop if

A) it rotates around the axis A.

B) it rotates around the axis A and the connections P and Q are briged.

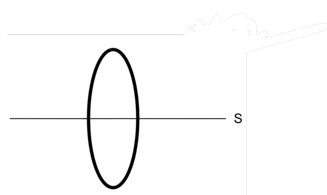
C) the current I is decreased slowly.

D) the conductor loop is moved one third of its width to the right.

E) the magnetic flux through the condctor loop changes.

Question 1.15 (MC)

An electrical charge of $3.25\,\mu C$ is evenly distributed on a ring of radius $7.5\,cm$. The cross-section of the ring is negligibly small. What is the magnitude of the electric field on the symmetry axis S at a distance $1.2\,cm$ from the center of the ring?

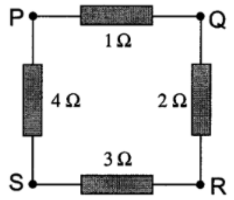


A) $52\,mN \cdot C^{-1}$ B) $80\,N \cdot C^{-1}$ C) $27\,kN \cdot C^{-1}$

D) $85\,kN \cdot C^{-1}$ E) $8 \times 10^{5}\,N \cdot C^{-1}$

Question 1.16 (MC)

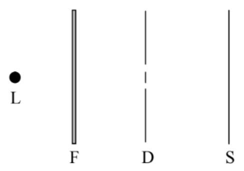
Four resistors are connected as shown below. Between which pair of points is the resistance the largest?



A) P and Q B) Q and S C) R and S
D) S and P E) P and R

Question 1.17 (MC)

The sketch below shows an interference experiment using a double slit. The drawing is not to scale. The light source L emits white light, the filter F absorbs anything but the green part of the light. The filtered light passes through the double slit D and creates an interference pattern on the screen S. The interference pattern consists of almost equidistant light and dark fringes.



Which of the following actions will reduce the distance between the interference fringes?

1. Replacing the filter F by a filter which only allows blue light to pass through.
2. Replacing the double slit by another double slit with a larger distance between the slits.
3. Using a light source with larger luminosity.

A) All three B) Only 1 and 2
C) Only 2 and 3 D) Only 1
E) Only 3

Question 1.18 (MC)

Cepheids are a special type of star with a periodic change in luminosity. Knowing the period of this oscillation, one can deduce the absolute luminosity. You have a camera with which you can measure the intensity of starlight. You take pictures of two Cepheids A and B and find that the measured intensity of Cepheid B is 18 times weaker than that of Cepheid A. From their respective periods you deduce that the absolute luminosity of Cepheid A is twice that of Cepheid B. How much farther away is Cepheid B than Cepheid A?

A) 3 times as far
B) 6 times as far
C) 9 times as far
D) 18 times as far
E) 36 times as far

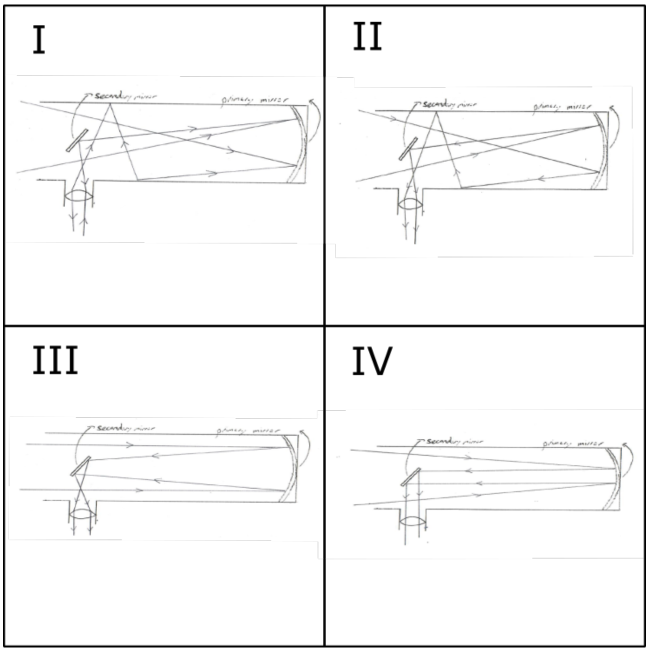
Question 1.19 (MC)

How can one increase the amount of photons received by a telescope?

A) Having a telescope with a bigger primary mirror.
B) Having a telescope with a bigger secondary mirror.
C) Having a telescope with a bigger eyepiece.
D) Instead of using an eyepiece, one can directly attach a sensitive camera.
E) Filtering out the red light and only keeping the high-energy blue photons.

Question 1.20 (MC)

Consider a telescope. Which of the following light paths is possible?



A) I B) II C) III D) IV

Question 1.21 (MC)

Emmy holds a spoon an arm length away and looks at her reflection in both the inside and the outside of the spoon. In doing so, she notices the following:

A) The mirror images on the inside and outside are reversed (upside down).

B) The mirror image on the inside is reversed and the mirror image on the outside is not reversed.

C) The mirror image on the outside is reversed and the mirror image on the inside is not reversed.

D) The mirror images on the inside and outside are both not reversed.

Multiple Choice: answer sheet

Indicate your answers in the corresponding boxes on this page.

Last name:	**First name:**	**Total:**

	A	B	C	D	E
Question 1.1	☐	☐	☐	☐	
Question 1.2	☐	☐	☐	☐	☐
Question 1.3	☐	☐	☐	☐	☐
Question 1.4	☐	☐	☐		
Question 1.5	☐	☐	☐	☐	☐
Question 1.6	☐	☐	☐	☐	
Question 1.7	☐	☐	☐	☐	☐
Question 1.8	☐	☐	☐	☐	☐
Question 1.9	☐	☐	☐	☐	☐
Question 1.10	☐	☐	☐	☐	
Question 1.11	☐	☐	☐	☐	☐
Question 1.12	☐	☐	☐	☐	☐
Question 1.13	☐	☐	☐	☐	
Question 1.14	☐	☐	☐	☐	☐
Question 1.15	☐	☐	☐	☐	☐
Question 1.16	☐	☐	☐	☐	☐
Question 1.17	☐	☐	☐	☐	☐
Question 1.18	☐	☐	☐	☐	☐
Question 1.19	☐	☐	☐	☐	☐
Question 1.20	☐	☐	☐	☐	
Question 1.21	☐	☐	☐	☐	

Multiple Choice: solutions

	A	B	C	D	E
Question 1.1	■	☐	☐	☐	
Question 1.2	☐	■	☐	☐	☐
Question 1.3	☐	■	☐	☐	☐
Question 1.4	■	☐	☐		
Question 1.5	☐	■	☐	☐	☐
Question 1.6	☐	☐	■	☐	
Question 1.7	☐	☐	☐	■	☐
Question 1.8	☐	☐	☐	■	☐
Question 1.9	■	☐	☐	☐	☐
Question 1.10	■	☐	☐	☐	
Question 1.11	☐	☐	■	☐	☐
Question 1.12	☐	☐	☐	■	☐
Question 1.13	■	☐	☐	☐	
Question 1.14	☐	☐	☐	■	☐
Question 1.15	☐	☐	☐	☐	■
Question 1.16	☐	■	☐	☐	☐
Question 1.17	☐	■	☐	☐	☐
Question 1.18	■	☐	☐	☐	☐
Question 1.19	■	☐	☐	☐	☐
Question 1.20	☐	☐	■	☐	
Question 1.21	☐	■	☐	☐	

Long problems

Duration: 120 minutes
Marks: 48 points (3×16)

Start each problem on a new sheet in order to ease the correction.

General hint: The problems consist of partially independent problem parts, so if you get stuck, it is a good idea to read further ahead and to continue with an easier part.

Long problem 2.1: Magnus effect *(16 points)*

Rotating flying objects are deviated from their ballistic trajectory due to their interaction with the surrounding air flow. In this exercise we want to have a closer look at this phenomenon named after the physicist Heinrich Magnus. We will elaborate a simple model in part A in order to explain the effect and will then apply our insights to an example in part B.

Part A. Magnus effect *(9 points)*

In this section we will consider a cylinder with height h and radius R. The cylinder is rotating along its axis with an angular velocity ω and its center of gravity is moving with a velocity v_B. For part A we will set our frame of reference such that it is moving with velocity v_B along with the cylinder (see sketch).

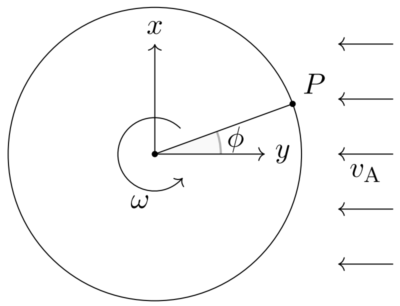


Figure A.1: Cylinder seen from the reference frame moving along with it

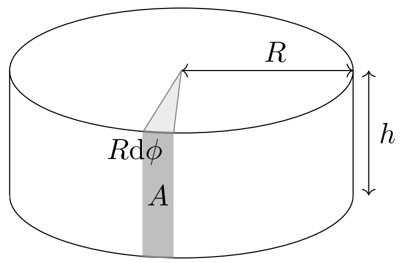
i. (1 pt) Derive an expression for v_x and v_y, the components of the velocity of point P on the edge of the cylinder in terms of ω, R, and θ.

ii. (0.5 pts) What is the velocity, $\vec{v}_A$ of the air far in front of the cylinder?

The layer of air at the edge of the cylinder is swept along due to the rotation of the cylinder. In consequence, we assume that the velocity of the air at point P is the sum of $\vec{v}_A$ and $\vec{v}_P$, the velocity of point P on the cylinder.

iii. (3 pts) Granted that the pressure at point $P_0 = (R, 0)$ is p_0, use Bernoulli's equation in order to deduce the pressure at an arbitrary point P on the edge of the cylinder in terms of v_B, the angle ϕ, ω and R.

We now consider an infinitesimal area element on the side of the cylinder.



iv. (1.5 pts) Compute the x and y component of the force which is acting on surface A with length $R\mathrm{d}\phi$ and height h.

v. (1 pt) In which direction points the total force on the cylinder? An argument without calculations is sufficient.

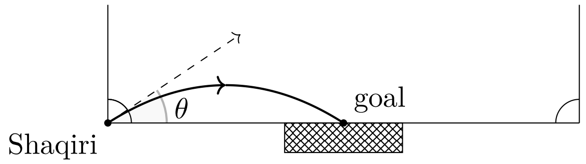
vi. (2 pts) What is the magnitude of that force?
Hint: $\int_0^{2\pi} \sin(\phi)^2 \,\mathrm{d}\phi = \pi$

Part B. Trajectory *(7 points)*

In this part we will apply the Magnus effect on the trajectory of a football. We therefore need the Magnus force in case of a ball flying with velocity v and rotating with an angular velocity ω

$$\vec{F} = \frac{4}{3}\pi R^3 \rho_A \left(\vec{v} \times \vec{\omega}\right),$$

where ρ_A is the density of air ($1.2\,\mathrm{kg}\cdot\mathrm{m}^{-3}$) and R the radius of the ball. Moreover, we assume for this exercise that the football weighs 420 g and has a radius of 11 cm. The distance from the corner flag to the center of the goal measures $L = 23\,\mathrm{m}$. Shaqiri wants to score a goal from the corner. He therefore shoots the ball such that its axis of rotation always remains perpendicular to the ground.



i. (2 pts) What shapes do the horizontal and vertical components of the ball's trajectory describe?

ii. (2.5 pts) Shaqiri hits the ball with a rotation rate of 10 rotations per second and an initial horizontal speed of $v_h = 80\,\mathrm{km}\cdot\mathrm{h}^{-1}$. What angle θ with respect to the base line does Shaqiri have to aim so that the ball crosses the goal line at its very center?

iii. (2.5 pts) With what vertical upwards speed does Shaqiri have to hit the ball so that it touches the ground again right at the moment when it crosses the goal line?

Long problem 2.2: Energy crisis *(16 points)*

On a lovely autumn day, Richard Feynman reads in the newspaper that a fossil fuel shortage is expected during the coming winter. He currently lives together with Arline Greenbaum in a shared flat heated on oil. They start wondering how to reduce their oil consumption. During winter, the outside temperature is $T_1 = 3\,°\mathrm{C}$ and they want their flat to be kept at $T_2 = 20\,°\mathrm{C}$, in which case the flat loses $P = 2000\,\mathrm{W}$ of heat that needs to be compensated. Fuel oil has a heating value of $H = 36\,\mathrm{MJ \cdot L^{-1}}$. Water has a heat capacity of $c = 4.19\,\mathrm{kJ \cdot kg^{-1} \cdot K^{-1}}$ and a latent heat of solidification of $L = 333.7\,\mathrm{kJ \cdot kg^{-1}}$.

Part A. Warm-up questions *(4.5 points)*

i. (1 pt) Estimate the efficiency of heating up domestic hot water with fuel oil.

ii. (1 pt) How much water could be heated from T_2 to $T_3 = 80\,°\mathrm{C}$ with $1\,\mathrm{L}$ of fuel oil?

iii. (0.5 pts) How long can they maintain their flat's temperature with $1\,\mathrm{L}$ of oil?

iv. (2 pts) Assuming that the flat only loses heat through conduction, how much power would they save by reducing the flat's temperature down to $T_2' = 17\,°\mathrm{C}$?

Part B. For a few heating systems more *(11.5 points)*

For comfort, they don't want to reduce the temperature. So instead, they consider other heating systems and calculate the ressources needed to maintain their flat's temperature.

i. (1 pt) They could buy an electric heater. How much electric power would they then need?

They could also install a heat pump working between two different temperatures $T_a \leq T_b$, powered by an electric motor.

ii. (3 pts) Assuming that the heat pump is as efficient as theoretically possible, find a relation between an infinitesimal amount of supplied work $\mathrm{d}W$ and an infinitesimal amount of heat $\mathrm{d}Q_b$ transferred to the higher temperature reservoir.

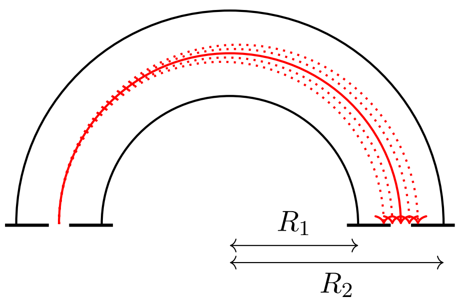
iii. (2 pts) The most straightforward way to install a heat pump is to make it work between the air from outside and from the flat. What would be the theoretical minimum electric power needed in this case? You can assume that the electric motor has $100\,\%$ efficiency, because most of its energy losses directly contribute to the flat's temperature.

iv. (4.5 pts) Richard then thinks about their freezer, which also works like a heat pump. He realizes that they could take tap water (coming into the house at temperature T_2), put it in a bucket in the freezer and, as soon as it is frozen, throw the resulting ice out the window and repeat the process. What would then be the theoretical minimum average electric power needed, and how much water would be required per hour in average? Assume again that the freezer's pump motor has $100\,\%$ efficiency and that opening the window when throwing out the ice doesn't impact the room temperature, nor does the thrown ice influence the outside temperature.
Hint: consider some input water of mass m and treat the process of lowering the water temperature and the process of freezing separately at first. In the end, assume that both processes happen in a timespan Δt to get average values.

v. (1 pt) Suddenly the doorbell rings. Marie visits Arline and Richard and brings them the long-promised bike hometrainer. Could Arline and Richard generate the needed heat just by training enough?

Long problem 2.3: Hemispherical analyzer *(16 points)*

The aim of photoelectron spectroscopy is the examination of solids on their electrical properties by irradiating the material in question with light so as to determine the energy of the ejected electrons. In order to measure the energy of the electrons, a hemispherical analyzer is used (see drawing below). The underlaying principle of the measurement is that depending on its kinetic energy an electron is deviated differently by the electric field between two conductive hemispheres. In other words, the determination of the electrons position after its flights through the hemispherical analyzer allows to conclude on its initial kinetic energy. We now want to find out in more details how this works.



Part A. Electric Field *(7 points)*

In this section we want to compute the electric field between the two hemispheres. We therefore assume that the electric field between the hemispheres is the same as it would be between two nested spheres with radii R_1 and R_2.

i. (1 pt) First, we assume that the inner sphere with radius R_1 is charged with a charge Q_1. What is the electric field caused by that charge in the interspace between the two spheres in dependence on the radius r.

ii. (1 pt) There is also a charge Q_2 on the bigger sphere with radius R_2. What is the electric field caused by that charge in the space between the spheres in dependence on the radius r.

iii. (1 pt) What is the electric potential between the two spheres in terms of r, Q_1, and Q_2? You can set the reference $V = 0$ for the potential wherever you want to.

iv. (3 pts) Since we cannot directly influence the charges Q_1 and Q_2 in a laboratory but only the potentials V_1 and V_2 of the two spheres, express the potential in terms of V_1, V_2, R_1, R_2, and r.

v. (1 pt) What is the magnitude of the electric field expressed in terms of V_1, V_2, R_1, R_2 and r?

Part B. Electron Orbit *(9 points)*

We assume that the electrons enter the hemispherical analyzer at a radius $R_\mathrm{i} = \frac{1}{2}(R_1 + R_2)$ with a velocity perpendicular to the opening.

i. (2.5 pts) What kinetic energy $E_\mathrm{kin}^\mathrm{p}$ does an electron need to have such that it describes a circular orbit? Express the result in terms of V_1, V_2, R_1, R_2, and the elementary charge e.

ii. (1 pt) We now consider electrons for which $E_\mathrm{kin} \neq E_\mathrm{kin}^\mathrm{p}$. What is the shape of their trajectory inside the analyzer in that case?

iii. (5.5 pts) Determine the radius R_f of an electron at the exit of the hemispherical analyzer depending on its kinetic energy at the entrance E_kin, the pass energy $E_\mathrm{kin}^\mathrm{p}$, and R_i. *Hint: You may want to make use of conservation of angular momentum and conservation of energy.*

Long problems: solutions

Long problem 2.1: Magnus effect — 16

Rotating flying objects are deviated from their ballistic trajectory due to their interaction with the surrounding air flow. In this exercise we want to have a closer look at this phenomenon named after the physicist Heinrich Magnus. We will elaborate a simple model in part A in order to explain the effect and will then apply our insights to an example in part B.

Part A. Magnus effect — 9

In this section we will consider a cylinder with height h and radius R. The cylinder is rotating along its axis with an angular velocity ω and its center of gravity is moving with a velocity $v_\mathbf{B}$. For part A we will set our frame of reference such that it is moving with velocity $v_\mathbf{B}$ along with the cylinder (see sketch).

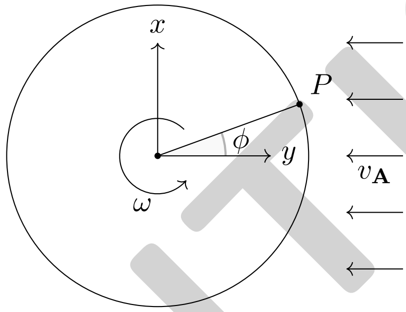


Figure A.1: Cylinder seen from the reference frame moving along with it

i. Derive an expression for v_x and v_y, the components of the velocity of point P on the edge of the cylinder in terms of ω, R, and θ. — *1*

$$v_x = -\sin(\phi)\,\omega R$$

0.5

$$v_y = \cos(\phi)\,\omega R$$

0.5

ii. What is the velocity, $\vec{v}_\mathbf{A}$ of the air far in front of the cylinder? — *0.5*

$$\vec{v}_\mathrm{A} = (-v_\mathrm{B}, 0)$$

0.5

The layer of air at the edge of the cylinder is swept along due to the rotation of the cylinder. In consequence, we assume that the velocity of the air at point P is the sum of $\vec{v}_\mathbf{A}$ and $\vec{v}_P$, the velocity of point P on the cylinder.

iii. Granted that the pressure at point $P_0 = (R, 0)$ is p_0, use Bernoulli's equation in order to deduce the pressure at an arbitrary point P on the edge of the cylinder in terms of v_B, the angle ϕ, ω and R. ***3***

The velocity of the air around the cylinder is

$$(v_x, v_y) = (-v_\mathrm{B} - \sin(\phi)\,\omega R, \cos(\phi)\,\omega R)\,.$$

0.5

The Bernoulli equation reads as

$$\frac{1}{2}\rho_\mathrm{A} v_0^2 + p_0 = \frac{1}{2}\rho_\mathrm{A}\left(v_x^2 + v_y^2\right) + p(\phi)\,,$$

where we neglected the hydrostatic pressure. 1

With the previous result the velocities can be expanded

$$v_0^2 = v_\mathrm{B}^2 + (\omega R)^2$$

0.5

$$v_x^2 + v_y^2 = v_\mathrm{B}^2 + (\omega R)^2 + 2\sin(\phi)\,v_\mathrm{B}\omega R$$

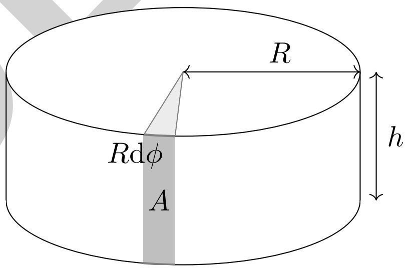
0.5

Putting all things together we get

$$p(\phi) = p_0 - \sin(\phi)\,v_\mathrm{B}\omega R\rho_\mathrm{A}.$$

0.5

We now consider an infinitesimal area element on the side of the cylinder.



iv. Compute the x and y component of the force which is acting on surface A with length $R\mathrm{d}\phi$ and height h. ***1.5***

The total force is given by

$$F = pA.$$

0.5

Therefore we get

$$F_x = -F\cos(\phi) = -\left(p_0 - \rho_\mathrm{A}\sin(\phi)\, v_\mathrm{B}\omega R\right)\cos(\phi)\, Rh\mathrm{d}\phi$$

0.5

and

$$F_y = -F\sin(\phi) = -\left(p_0 - \rho_\mathrm{A}\sin(\phi)\, v_\mathrm{B}\omega R\right)\sin(\phi)\, Rh\mathrm{d}\phi.$$

0.5

v. In which direction points the total force on the cylinder? An argument without calculations is sufficient. ***1***

If we consider the two points (x, y) and $(-x, y)$ the x component of the pressure force is the same in magnitude but the opposite in direction. Therefore the x component is zero. 0.5

Since the pressure on the upper side of the cylinder (positive y values) is lower the than on the lower side, the force will point in positive y-direction. 0.5

vi. What is the magnitude of that force?
Hint: $\int_0^{2\pi}\sin(\phi)^2\,\mathrm{d}\phi = \pi$ ***2***

From the argument above we only have to consider the y component of the force. We get the total force by integration:

$$F = \int_0^{2\pi} F_y\mathrm{d}\phi = -\int_0^{2\pi}\left(p_0 - 2\sin(\phi)\, v_\mathrm{B}\omega R\right)\sin(\phi)\, Rh\mathrm{d}\phi.$$

0.5

The term proportional to the sine becomes zero:

$$-\int_0^{2\pi} p_0\sin(\phi)\, Rh\mathrm{d}\phi = 0.$$

0.5

And for the other term we can use the hint to get

$$F = \int_0^{2\pi} F_y\mathrm{d}\phi = \int_0^{2\pi}\rho_\mathrm{A}\sin(\phi)^2\, v_\mathrm{B}\omega R^2 h\mathrm{d}\phi = v_\mathrm{B}\omega\pi R^2 h = \omega v_\mathrm{B} V_\mathrm{cylinder}\rho_\mathrm{A}.$$

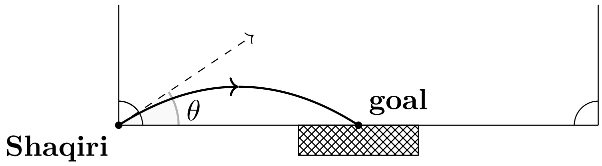
1

Part B. Trajectory *7*

In this part we will apply the Magnus effect on the trajectory of a football. We therefore need the Magnus force in case of a ball flying with velocity v and rotating with an angular velocity ω

$$\vec{F} = \frac{4}{3}\pi R^3\rho_\mathbf{A}\left(\vec{v}\times\vec{\omega}\right),$$

where $\rho_\mathbf{A}$ is the density of air $(1.2\,\mathrm{kg}\cdot\mathrm{m}^{-3})$ and R the radius of the ball. Moreover, we assume for this exercise that the football weighs $420\,\mathrm{g}$ and has a radius of $11\,\mathrm{cm}$. The distance from the corner flag to the center of the goal measures $L = 23\,\mathrm{m}$. Shaqiri wants to score a goal from the corner. He therefore shoots the ball such that its axis of rotation always remains perpendicular to the ground.



i. What shapes do the horizontal and vertical components of the ball's trajectory describe? ***2***

The two components can be considered independently, because the rotation is along the vertical axis and is therefore only affecting the horizontal movement. 0.5

For the horizontal movement we just have the force from the Magnus-effect, which has the same form as the Lorenz force, therefore we expect that the ball moves on a circular trajectory. 1

For the vertical movement only gravity plays a role, therefore we get a parabola. 0.5

ii. Shaqiri hits the ball with a rotation rate of 10 rotations per second and an initial horizontal speed of $v_\mathrm{h} = 80\,\mathrm{km}\cdot\mathrm{h}^{-1}$. What angle θ with respect to the base line does Shaqiri have to aim so that the ball crosses the goal line at its very center? ***2.5***

The magnus force contributes the centripetal force thereby we have the condition

$$\frac{m_\mathrm{B} v_\mathrm{h}^2}{R_\mathrm{f}} = m_\mathrm{A} v_\mathrm{h} \omega,$$

where ω is the angular velocity of the ball's rotation and R_f the radius of the trajectory. 0.5

Solving for R_f gives

$$R_\mathrm{f} = \frac{m_\mathrm{B}}{m_\mathrm{A}} \frac{v_\mathrm{h}}{\omega}.$$

0.5

The opening angle ϕ of the arc describing the ball's trajectory needs to fullfil the condition

$$\sin\left(\frac{\phi}{2}\right) R_\mathrm{f} = \frac{L}{2}.$$

0.5

Shaqiri needs to aim at half of this opening angle, therefore

$$\theta = \arcsin\left(\frac{L}{2R_\mathrm{f}}\right) = \arcsin\left(\frac{L\omega}{2v_\mathrm{h}} \frac{m_\mathrm{A}}{m_\mathrm{B}}\right).$$

0.5

The numerical answer is $31.2°$ (the radius is $22\,\mathrm{m}$). 0.5

iii. With what vertical upwards speed does Shaqiri have to hit the ball so that it touches the ground again right at the moment when it crosses the goal line? ***2.5***

Let $v_{p,0}$ the initial vertical velocity of the ball then the vertical velocity of the ball in dependence on time is

$$v_p(t) = v_{p,0} + gt.$$

0.5

When the ball hits the ground it has a velocity of $-v_{p,0}$, 0.5

which means $v_{p,0} = \frac{gT}{2}$, where T is the time it takes the Ball to hit the goal. 0.5

The time T can be obtained from the previous task

$$T = \frac{2\theta R}{v_\mathrm{h}}.$$

0.5

The numerical answer is $v_{p,0} = 5.33\,\mathrm{m}\cdot\mathrm{s}^{-1}$ ($T = 1.09\,\mathrm{s}$). 0.5

Long problem 2.2: Energy crisis *16*

On a lovely autumn day, Richard Feynman reads in the newspaper that a fossil fuel shortage is expected during the coming winter. He currently lives together with Arline Greenbaum in a shared flat heated on oil. They start wondering how to reduce their oil consumption. During winter, the outside temperature is $T_1 = 3\,°\mathrm{C}$ and they want their flat to be kept at $T_2 = 20\,°\mathrm{C}$, in which case the flat loses $P = 2000\,\mathrm{W}$ of heat that needs to be compensated. Fuel oil has a heating value of $H = 36\,\mathrm{MJ \cdot L^{-1}}$. Water has a heat capacity of $c = 4.19\,\mathrm{kJ \cdot kg^{-1} \cdot K^{-1}}$ and a latent heat of solidification of $L = 333.7\,\mathrm{kJ \cdot kg^{-1}}$.

Part A. Warm-up questions *4.5*

i. Estimate the efficiency of heating up domestic hot water with fuel oil. *1*

The efficiency is close to 100 %, the only losses are through the walls of the furnace and/or boiler. However, the longer the time between the heating and the use of domestic hot water, the higher the losses. 80 % is a safe value. 1

ii. How much water could be heated from T_2 to $T_3 = 80\,°\mathrm{C}$ with 1 L of fuel oil? *1*

Heating water from T_2 to T_3 requires a heat of $c\,(T_3 - T_2)$ per unit mass. 0.5

So with $l = 1\,\mathrm{L}$ of fuel oil, one can heat a mass of water of $\frac{lH}{c(T_3-T_2)} \approx 143\,\mathrm{kg}$, which also corresponds to $\approx 143\,\mathrm{L}$. Taking the efficiency into account, a value between 110 L and 143 L is considered correct. 0.5

iii. How long can they maintain their flat's temperature with 1 L of oil? *0.5*

They can maintain it for $\frac{lH}{P} = 1.8 \times 10^4\,\mathrm{s} = 5\,\mathrm{h}$. 0.5

iv. Assuming that the flat only loses heat through conduction, how much power would they save by reducing the flat's temperature down to $T_2' = 17\,°\mathrm{C}$? *2*

Heat transfer through conduction is proportional to the temperature difference. 1

Therefore the heat loss would be equal to $\frac{T_2'-T_1}{T_2-T_1}P$. 0.5

The power saved is then $P - \frac{T_2'-T_1}{T_2-T_1}P = \frac{T_2-T_2'}{T_2-T_1}P \approx 353\,\mathrm{W}$ or 18 %. 0.5

Part B. For a few heating systems more *11.5*

For comfort, they don't want to reduce the temperature. So instead, they consider other heating systems and calculate the ressources needed to maintain their flat's temperature.

i. They could buy an electric heater. How much electric power would they then need? *1*

Electric heaters have near-perfect efficiency, because what would be losses in other systems are the actual useful output. So they would need a power $P = 2000\,\mathrm{W}$. 1

They could also install a heat pump working between two different temperatures $T_a \leq T_b$, powered by an electric motor.

ii. Assuming that the heat pump is as efficient as theoretically possible, find a relation between an infinitesimal amount of supplied work $\mathrm{d}W$ and an infinitesimal amount of heat $\mathrm{d}Q_b$ transferred to the higher temperature reservoir. *3*

The ideal case corresponds to a Carnot cycle where the total entropy variation is null and the internal energy of the heat pump goes back to its initial value after each cycle. 1

To function, the pump requires the work $\mathrm{d}W = \mathrm{d}Q_b - \mathrm{d}Q_a$ to be supplied, where $\mathrm{d}Q_a$ is taken positive (heat transferred from the lower temperature reservoir to the pump). 0.5

From entropy conservation, $\frac{\mathrm{d}Q_a}{T_a} = \frac{\mathrm{d}Q_b}{T_b}$. 0.5

Therefore, $\mathrm{d}W = \mathrm{d}Q_b - \mathrm{d}Q_b \frac{T_a}{T_b} = \mathrm{d}Q_b \frac{T_b - T_a}{T_b}$. 1

iii. The most straightforward way to install a heat pump is to make it work between the air from outside and from the flat. What would be the theoretical minimum electric power needed in this case? You can assume that the electric motor has 100 % efficiency, because most of its energy losses directly contribute to the flat's temperature. *2*

We can use the first formula derived previously, with $T_a = T_1$ and $T_b = T_2$. 0.5

Because the pump motor is assumed to have perfect efficiency, $\frac{\mathrm{d}W}{\mathrm{d}t}$ is the required electric power. 0.5

We also have $P = \frac{\mathrm{d}Q_2}{\mathrm{d}t}$. 0.5

Therefore, $\frac{\mathrm{d}W}{\mathrm{d}t} = P \frac{T_2 - T_1}{T_2} \approx 116\,\mathrm{W}$. 0.5

iv. Richard then thinks about their freezer, which also works like a heat pump. He realizes that they could take tap water (coming into the house at temperature T_2), put it in a bucket in the freezer and, as soon as it is frozen, throw the resulting ice out the window and repeat the process. What would then be the theoretical minimum average electric power needed, and how much water would be required per hour in average? Assume again that the freezer's pump motor has 100 % efficiency and that opening the window when throwing out the ice doesn't impact the room temperature, nor does the thrown ice influence the outside temperature.
Hint: consider some input water of mass m and treat the process of lowering the water temperature and the process of freezing separately at first. In the end, assume that both processes happen in a timespan Δt to get average values. *4.5*

In this case, the upper temperature remains fixed $T_b = T_2$, whereas the lower temperature T_a drops from T_2 to $T_0 = 0\,°\mathrm{C}$, where water freezes.

When lowering the temperature, we have $\mathrm{d}Q_a = -cm\mathrm{d}T_a$. 0.5

Integrating it in temperature, we get

$$Q_{1,T_2 \to T_0} = cm \int_{T_0}^{T_2} T \mathrm{d}T = cm\,(T_2 - T_0)\,.$$

0.5

Integrating in temperature the entropy conservation equation, we get

$$Q_{2,T_2 \to T_0} = cm \int_{T_0}^{T_2} \frac{T_2}{T} \mathrm{d}T = cmT_2 \log\left(\frac{T_2}{T_0}\right).$$

0.5

During freezing, we have $Q_{1,\text{freezing}} = mL$. 0.5

So $Q_{2,\text{freezing}} = Q_{1,\text{freezing}} \frac{T_2}{T_0} = mL \frac{T_2}{T_0}$. 0.5

In total, $Q_2 = cmT_2 \log\left(\frac{T_2}{T_0}\right) + mL\frac{T_2}{T_0}$ and $W = cm\left(T_2 \log\left(\frac{T_2}{T_0}\right) + T_0 - T_2\right) + mL\frac{T_2 - T_0}{T_0}$. 0.5

The timespan Δt is such that $P = \frac{Q_2}{\Delta t}$. 0.5

Then

$$\frac{m}{\Delta t} = P \frac{1}{cT_2 \log\left(\frac{T_2}{T_0}\right) + L\frac{T_2}{T_0}} \approx 5.4 \times 10^{-4}\,\mathrm{kg \cdot s^{-1}},$$

which is about $2.0\,\mathrm{L \cdot h^{-1}}$. 0.5

And

$$\frac{W}{\Delta t} = P\left(1 - \frac{c\,(T_2 - T_0) + L}{cT_2 \log\left(\frac{T_2}{T_0}\right) + L\frac{T_2}{T_0}}\right) \approx 135\,\mathrm{W}.$$

0.5

v. Suddenly the doorbell rings. Marie visits Arline and Richard and brings them the long-promised bike hometrainer. Could Arline and Richard generate the needed heat just by training enough? ***1***

The daily food ration for an adult is around $10\,000\,\mathrm{kJ}$. 0.5

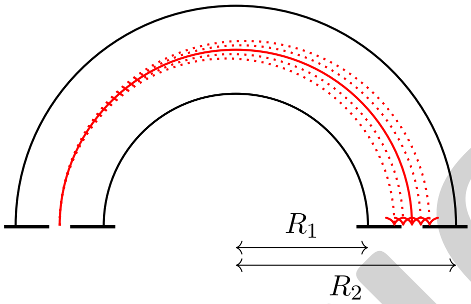
So even if Arline and Richard transformed all their food intake in heat, their combined power would be a mere

$$2\frac{10\,000\,\mathrm{kJ}}{3600 \cdot 24\,\mathrm{s}} \approx 231\,\mathrm{W},$$

well below P. 0.5

Long problem 2.3: Hemispherical analyzer *16*

The aim of photoelectron spectroscopy is the examination of solids on their electrical properties by irradiating the material in question with light so as to determine the energy of the ejected electrons. In order to measure the energy of the electrons, a hemispherical analyzer is used (see drawing below). The underlaying principle of the measurement is that depending on its kinetic energy an electron is deviated differently by the electric field between two conductive hemispheres. In other words, the determination of the electrons position after its flights through the hemispherical analyzer allows to conclude on its initial kinetic energy. We now want to find out in more details how this works.



Part A. Electric Field *7*

In this section we want to compute the electric field between the two hemispheres. We therefore assume that the electric field between the hemispheres is the same as it would be between two nested spheres with radii R_1 and R_2.

i. First, we assume that the inner sphere with radius R_1 is charged with a charge Q_1. What is the electric field caused by that charge in the interspace between the two spheres in dependence on the radius r. *1*

The electric field is

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}$$

1

ii. There is also a charge Q_2 on the bigger sphere with radius R_2. What is the electric field caused by that charge in the space between the spheres in dependence on the radius r. *1*

The electric field is $E(r) = 0$ 1

iii. What is the electric potential between the two spheres in terms of r, Q_1, and Q_2? You can set the reference $V = 0$ for the potential wherever you want to. *1*

We integrate the total electric field with respect to r and get

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r} + C_2$$

1

iv. Since we cannot directly influence the charges Q_1 and Q_2 in a laboratory but only the potentials V_1 and V_2 of the two spheres, express the potential in terms of V_1, V_2, R_1, R_2, and r. *3*

We have the two boundary conditions

$$V_1 = V(R_1) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1} + C_2$$

0.5

$$V_2 = V(R_2) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_2} + C_2$$

0.5

This can be solved for

$$\frac{Q_1}{4\pi\varepsilon_0} = -\frac{V_2 - V_1}{R_2 - R_1}R_2R_1$$

1

and

$$C_2 = V_1 + \frac{V_2 - V_1}{R_2 - R_1}R_2 = \frac{V_2R_2 + V_1R_1}{R_2 - R_1}$$

1

v. What is the magnitude of the electric field expressed in terms of V_1, V_2, R_1, R_2 and r? *1*

We take the derivative of the Potential with respect to r and get

$$|E(r)| = \frac{V_2 - V_1}{R_2 - R_1}\frac{R_2R_1}{r^2}$$

1

Part B. Electron Orbit *9*

We assume that the electrons enter the hemispherical analyzer at a radius $R_\mathrm{i} = \frac{1}{2}(R_1 + R_2)$ with a velocity perpendicular to the opening.

i. What kinetic energy $E_{\mathrm{kin}}^{\mathrm{p}}$ does an electron need to have such that it describes a circular orbit? Express the result in terms of V_1, V_2, R_1, R_2, and the elementary charge e. *2.5*

The centripetal force required to put the electron on circular orbit is

$$F_\mathrm{c} = m_\mathrm{e}\frac{v_\mathrm{p}^2}{R_\mathrm{p}}$$

0.5

which has to be equal to electric force

$$eE(R_\mathrm{i}) = m_\mathrm{e}\frac{v_\mathrm{p}^2}{R_\mathrm{p}}$$

0.5

The electron kinetic energy is given by

$$E_{\mathrm{kin}}^{\mathrm{p}} = \frac{1}{2} m_{\mathrm{e}} v_{\mathrm{p}}^{2}$$

0.5

Therefore we can solve E_{p}

$$E_{\mathrm{kin}}^{\mathrm{p}} = \frac{1}{2} e E(R_{\mathrm{i}})\, R_{\mathrm{i}} = -\frac{1}{2} e \frac{V_2 - V_1}{R_2 - R_1} \frac{R_2 R_1}{R_i} = e\,(V_2 - V_1)\, \frac{R_1 R_2}{R_1^2 - R_2^2}$$

1

ii. We now consider electrons for which $E_{\mathrm{kin}} \neq E_{\mathrm{kin}}^{\mathrm{p}}$. What is the shape of their trajectory inside the analyzer in that case? ***1***

We have the $1/r$ Potential as in the case of celestial orbits. Therefore we can use Kepler's laws and we get that the trajectory is shaped like an ellipse. 1

Note that the Perihel and the Aphel of the orbit are at the entry or at the exit of the analyzer.

iii. Determine the radius R_{f} of an electron at the exit of the hemispherical analyzer depending on its kinetic energy at the entrance E_{kin}, the pass energy $E_{\mathrm{kin}}^{\mathrm{p}}$, and R_{i}. *Hint: You may want to make use of conservation of angular momentum and conservation of energy.* ***5.5***

Let v_{i} be the initial velocity at the entry and v_{f} the velocity at the exit. First we have to note that the orbit is perpendicular to the radius at the entry and therefore also has to be perpendicular at the exit. 1

Therefore angular momentum conservation reads as

$$m_{\mathrm{e}} R_{\mathrm{i}} v_{\mathrm{i}} = m_{\mathrm{e}} R_{\mathrm{f}} v_{\mathrm{f}}$$

1

Similarly we can use the energy conservation condition

$$E + \frac{\alpha}{R_{\mathrm{i}}} = E_{\mathrm{f}} + \frac{\alpha}{R_{\mathrm{f}}}$$

1

From the angular momentum conservation we can express

$$E_{\mathrm{f}} = \frac{R_{\mathrm{i}}^2}{R_{\mathrm{f}}^2} E$$

0.5

Therefore the energy conservation gives a quadratic equation in R_{f}

$$\left(E + \frac{\alpha}{R_{\mathrm{i}}}\right) R_{\mathrm{f}}^2 - \alpha R_{\mathrm{f}} - R_{\mathrm{i}}^2 E = 0$$

0.5

The solution to this quadratic equation is

$$R_{\mathrm{f}} = \frac{\alpha \pm \sqrt{\alpha^2 + 4R_{\mathrm{i}}^2 E^2 + 4\alpha R_{\mathrm{i}} E}}{2\left(E + \frac{\alpha}{R_{\mathrm{i}}}\right)} = \frac{\alpha \pm (\alpha + 2R_{\mathrm{i}} E)}{2\left(E + \frac{\alpha}{R_{\mathrm{i}}}\right)} = -R_{\mathrm{i}} \frac{E}{E + \frac{\alpha}{R_{\mathrm{i}}}}$$

0.5

The other solution with the minus just gives $R_f = R_i$, which corresponds to a full orbit.

From above we have $\frac{\alpha}{R_{\mathrm{i}}} = -eE(R_{\mathrm{i}})\, R_{\mathrm{i}} = -2E_{\mathrm{p}}$ 0.5

which gives the final result

$$R_{\mathrm{f}} = R_{\mathrm{i}} \frac{1}{2\frac{E_{\mathrm{p}}}{E} - 1}$$

0.5