PHYSICS.
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Physics Olympiad

Second Round

online, 19 January 2021

Part 1	**:**	**3 problems**
Duration	:	120 minutes
Total	:	48 points (3×16)
Authorized material	:	Calculator without database
		Writing and drawing material
		One A4 double-sided handwritten page of notes

Good luck!

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Natural constants

Caesium hyperfine frequency	$\Delta\nu_{\mathrm{Cs}}$	9.192 631 770	$\times 10^{9}$	s^{-1}
Speed of light in vacuum	c	2.997 924 58	$\times 10^{8}$	$\mathrm{m} \cdot \mathrm{s}^{-1}$
Planck constant	h	6.626 070 15	$\times 10^{-34}$	$\mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-1}$
Elementary charge	e	1.602 176 634	$\times 10^{-19}$	$\mathrm{A} \cdot \mathrm{s}$
Boltzmann constant	k_{B}	1.380 649	$\times 10^{-23}$	$\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-2}$
Avogadro constant	N_{A}	6.022 140 76	$\times 10^{23}$	mol^{-1}
Luminous efficacy of radiation	K_{cd}	6.83	$\times 10^{2}$	$\mathrm{cd} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{3} \cdot \mathrm{sr}$
Magnetic constant	μ_0	1.256 637 062 12(19)	$\times 10^{-7}$	$\mathrm{A}^{-2} \cdot \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$
Electric constant	ε_0	8.854 187 812 8(13)	$\times 10^{-12}$	$\mathrm{A}^{2} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-3} \cdot \mathrm{s}^{4}$
Gas constant	R	8.314 462 618...		$\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{mol}^{-1} \cdot \mathrm{s}^{-2}$
Stefan-Boltzmann constant	σ	5.670 374 419...	$\times 10^{-8}$	$\mathrm{K}^{-4} \cdot \mathrm{kg} \cdot \mathrm{s}^{-3}$
Gravitational constant	G	6.674 30(15)	$\times 10^{-11}$	$\mathrm{kg}^{-1} \cdot \mathrm{m}^{3} \cdot \mathrm{s}^{-2}$
Electron mass	m_{e}	9.109 383 701 5(28)	$\times 10^{-31}$	kg
Neutron mass	m_{n}	1.674 927 498 04(95)	$\times 10^{-27}$	kg
Proton mass	m_{p}	1.672 621 923 69(51)	$\times 10^{-27}$	kg
Standard acceleration of gravity	g_{n}	9.806 65		$\mathrm{m} \cdot \mathrm{s}^{-2}$

Theoretical Problems

Duration: 120 minutes
Marks: 48 points (3×16)

Start each problem on a new sheet in order to ease the correction.
Label the sheets with your name and the number of the problem. Furthermore, number your sheets.

General hint: The problems consist of partially independent problem parts, so if you get stuck, it is a good idea to read further ahead and to continue with an easier part.

Problem 1.1: Model of inelastic collision *(16 points)*

In this problem we will consider a model of an inelastic collision. The system (shown in figure 1.1.1) consists of three bodies A (of mass M), B and C (both of mass m), where B and C are connected by a spring with spring constant k. The body A will initially be moving with velocity V and will eventually collide with the initially stationary system B+C.

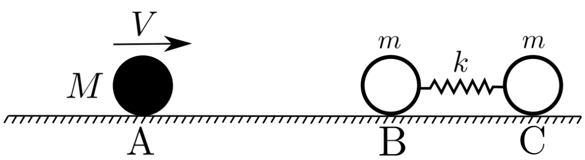


Figure 1.1.1: Setup of the problem.

Part A. First contact *(5 points)*

First we examine the *elastic* collision between the bodies A and B.

i. (1 pt.) What conserved quantities are relevant for such a collision?

ii. (3 pt.) What is the instantaneous velocity V' of body A right after the collision?

iii. (1 pt.) What is the instantaneous velocity v_1 of body B right after the collision?

Part B. In the center of mass frame *(5 points)*

From here on you can assume that the body A has no further interaction with the bodies B and C. Now we will analyze the motion of the B+C system. This system will undergo both translational motion and oscillatory motion. Thus, it is easiest to examine the B+C system in its center of mass reference frame.

i. (1 pt.) Find the velocity v_{CM} of the center of mass of the system B+C just after A and B made contact. This velocity of the center of mass will not change in the subsequent motion. Express your answer through v_1 and m.

ii. (1 pt.) What are the instantaneous velocities v_B^{CM} and v_C^{CM} of the bodies B and C in the center of mass reference frame at the moment when A and B make contact? Express your answer through v_1 and m.

iii. (3 pt.) At the moment of contact between A and B, the spring is in its equilibrium position. What is the energy E^{CM} and amplitude A (see figure 1.1.2) of the ensuing oscillations in the center of mass system? Express your answer in terms of v_1, m and k.

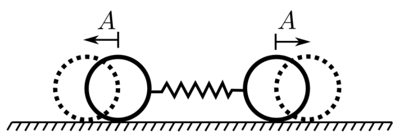


Figure 1.1.2: Definition of the amplitude of the oscillations.

Part C. Lost energy *(6 points)*

Going back to the reference frame that is fixed to the ground, we take a look at the collision between the body A and the composite system B+C with mass $2m$ and velocity v_{CM}. The total kinetic energy is the sum of the kinetic energy of body A and the kinetic energy of the composite system.

i. (2.5 pt.) What is the difference Q in the total kinetic energy before and after the collision? Express your answer in terms of V, m and M.

ii. (1 pt.) Prove that the “lost” energy Q is equal to the energy of the internal motion of the system B+C in the center of mass reference frame of B+C.

iii. (1 pt.) We define q as the ratio between the “lost” energy Q and the initial kinetic energy of A. Express q in terms of the ratio $\alpha = \frac{M}{m}$.

iv. (1.5 pt.) For which value of α is the fraction q the highest?

Problem 1.2: Condensed water vapor *(16 points)*

We consider two containers of 17.5 L filled with $n_0 = 1\,\text{mol}$ of water vapor at a pressure of 2 bar. We now want to find out how we can condense the water vapor to liquid water by cooling it. To perform the calculations, we assume in the following that water vapor can be described as an ideal gas with three degrees of freedom. In addition, the following measurements for water shall be given

- Molar mass of water: $18\,\text{g}\cdot\text{mol}^{-1}$
- Density of water: $1000\,\text{kg}\cdot\text{m}^{-3}$

For simplicity, assume that the density of water is independent of the pressure and temperature in the range that is relevant for this task. You are also given a separate document where a section of the p-T phase diagram for water is plotted.

Part A. Cooling *(8 points)*

i. (2 pt.) What is the temperature of the water vapor in the containers? Draw this initial state in the attached p-T diagram.

We now want to make the water vapor in the two containers condense. To do this, we cool one container isochorically and the other isobarically until they just begin to condense.

ii. (3 pt.) Draw these two processes in the attached p-T diagram.

iii. (1 pt.) At what pressure and temperature does the water vapor in the containers begin to condense?

iv. (2 pt.) Calculate how much heat was extracted from the water vapor in both processes. In which process less heat was extracted?

Part B. Phase transition *(6 points)*

In the following, we will only consider the container which we have cooled isobarically so far. We continue to cool this container isobarically in order to let the water vapor condense. This means that the pressure p_t remains constant and therefore also the temperature T_t is fixed by the vapor pressure curve during the phase transition. During the phase transition, the liquid and gaseous phases coexist. We denote the respective volumes by V_{li} and V_{gas}. We say that the phase transition is complete when all vapor has condensed to water.

i. (0.5 pt.) Let n_0 be the number of gas molecules before the phase transition begins and n_{gas} be the number of gas molecules during the phase transition. What is the number of molecules in the liquid phase?

ii. (1.5 pt.) What are the volumes V_{li} and V_{gas} at the beginning of the phase transition?

iii. (1.5 pt.) What are the volumes V_{li} and V_{gas} after the phase transition is complete?

iv. (1 pt.) Calculate the mechanical work exerted on the gas during the phase transition.

We now want to find out in more detail what happens during the phase transition.

v. (1.5 pt.) Find a formula for the volume of the system during the phase transition as a function of the instantaneous number of water vapor particles $n_{\text{gas}} < n_0$, the number of water vapor particles n_0 at the beginning of the phase transition, the pressure p_t, and the temperature T_t.

Part C. Isochoric phase transition *(2 points)*

i. (1 pt.) Describe qualitatively what happens if we cool isochorically instead of isobarically during the phase transition.

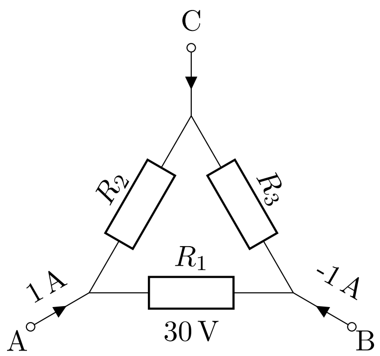
ii. (1 pt.) Draw this phase transition process in the p-T diagram.

Problem 1.3: Electric amplifier *(16 points)*

In many electrical circuits, signals have to be amplified. Be it the weak signal of an antenna or to turn the music louder. In this task we want to look at how one can set the gain of certain electrical amplifiers to a given value.

Part A. Warm-up problem *(2 points)*

To prepare ourselves for the following tasks, let us briefly review the two Kirchhoff rules. For this purpose we analyze the following circuit with three contacts A, B and C:



Based on measurements, we know that a current of 1 A flows into the circuit at contact A, that a current of 1 A flows out at contact B and that the voltage across resistor R_1 is 30 V.

i. (2 pt.) If the resistance R_3 is twice as large as R_2, what is the voltage across R_2?

Part B. Adjust the gain *(8 points)*

In this task we consider an inverting amplifier (see Fig. 1.3.1) and make the following assumptions:

- The input voltage and output voltage have different signs (hence the name "inverting amplifier").
- The input voltage U_{in} is amplified by a factor $k_0 = -10^6$.
- The amplifier is connected to a power supply with 12 V and −12 V (not drawn) and can produce output voltages only in this range.
- The input resistance (internal resistance between input and ground) into the amplifier is very large with $R_i = 10\,\mathrm{M\Omega}$.

i. (1 pt.) What is the maximum voltage range ΔU that can be applied at the input for the amplifier to still work properly?

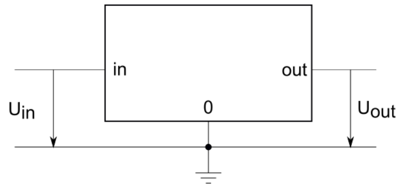


Figure 1.3.1: The rectangle symbolizes the amplifier, the output voltage U_{out} is the input voltage U_{in} amplified by k_0.

ii. (1 pt.) If a voltage of $U_{in} = 1\,\mu V$ is applied to the input, how much current flows through the input (in)?

iii. (2 pt.) By adding two resistors $R_1, R_2 \ll R_i$, the amplification can be set to any value $k < k_0$, see Fig. 1.3.2. What is the relationship (to a good approximation) between the currents I_1 and I_2?

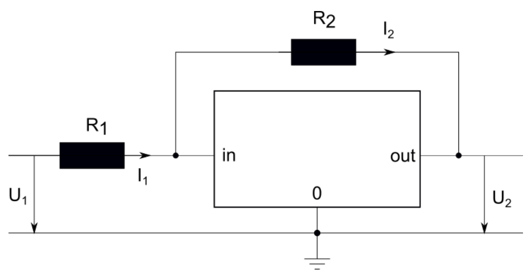


Figure 1.3.2: The amplifier with the two additional resistors R_1 and R_2. In addition, the applied voltage U_1 and the current I_1 through R_1, as well as analogously the voltage U_2 and current I_2 are drawn.

iv. (1.5 pt.) Suppose we want to achieve a gain $k \ll k_0$. What is the ratio between U_1 and U_2, resp. what is the resulting gain as a function of R_1 and R_2?

v. (1 pt.) What voltage U_{in} is applied to the amplifier?

vi. (1.5 pt.) Suppose you want to amplify a weak signal with an input voltage $U_1 = 20\,\mathrm{mV}$ by a factor $k = 100$ so that the signal has a maximum current of $10\,\mu A$. What values R_1 and R_2 do you choose? Does the approximation $R_1, R_2 \ll R_i$ still apply?

Part C. Slew rate *(6 points)*

We go back to the amplifier without additional resistors, see Fig. 1.3.1. Usually amplifiers cannot boost the input voltage arbitrarily fast, but need a certain time to react. The slew rate indicates the maximum rate at which the voltage at the output (out) of the amplifier can rise.

i. (2 pt.) Assume that the slew rate is $10\,\mathrm{V} \cdot \mathrm{\mu s}^{-1}$. What is approximately the maximum frequency that can still be amplified? Note: Assume that the amplitude of the output signal is about $1\,\mathrm{V}$ and the input signal is sinusoidal.

ii. (4 pt.) Suppose we immediately change the voltage at the input (in) from $0\,\mathrm{V}$ to $10\,\mathrm{\mu V}$ at $t = 0$. How does the output voltage look like (use numerical quantities given in this task)? Make a graph.

Theoretical Problems: solutions

Problem 1.1: Model of inelastic collision *16 pt.*

In this problem we will consider a model of an inelastic collision. The system (shown in figure 1.1.1) consists of three bodies A (of mass M), B and C (both of mass m), where B and C are connected by a spring with spring constant k. The body A will initially be moving with velocity V and will eventually collide with the initially stationary system B+C.

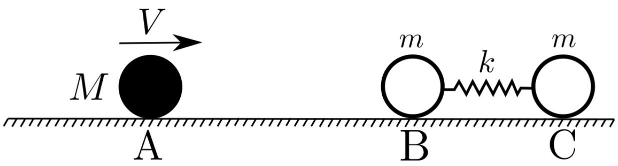


Figure 1.1.1: Setup of the problem.

Part A. First contact *5 pt.*

First we examine the *elastic* collision between the bodies A and B.

i. What conserved quantities are relevant for such a collision? **1 pt.**

The conserved quantities relevant for an elastic collision are

conservation of (linear) momentum and 0.5 pt.

conservation of *kinetic* energy. 0.5 pt.

No points should be given if it is only stated that the energy (instead of kinetic *energy) is conserved, except if the equation for conservation of kinetic energy is written in Part Aii.*

ii. What is the instantaneous velocity V' of body A right after the collision? **3 pt.**

As mentioned in the text, between the bodies A and B we have an elastic collision. This means that we have conservation of momentum

$$MV = MV' + mv_1, \tag{1.1.2}$$

1 pt.

and we also have conservation of energy

$$\frac{1}{2}MV^2 = \frac{1}{2}MV'^2 + \frac{1}{2}mv_1^2. \tag{1.1.3}$$

1 pt.

We can write this system of equations as

$$\begin{aligned} M(V - V')(V + V') &= mv_1^2 \\ M(V - V') &= mv_1 \end{aligned} \tag{1.1.4}$$

which then gives

$$v_1 = V + V'. \tag{1.1.5}$$

Plugging back into momentum conservation we have

$$V' = \frac{M-m}{M+m}V. \tag{1.1.6}$$

1 pt.

iii. What is the instantaneous velocity v_1 of body B right after the collision? **1 pt.**

Using the result of the previous section,

$$v_1 = V + V' = V + \frac{M-m}{M+m}V = \frac{2M}{M+m}V. \tag{1.1.7}$$

1 pt.

Part B. In the center of mass frame *5 pt.*

From here on you can assume that the body A has no further interaction with the bodies B and C. Now we will analyze the motion of the B+C system. This system will undergo both translational motion and oscillatory motion. Thus, it is easiest to examine the B+C system in its center of mass reference frame.

i. Find the velocity v_{CM} of the center of mass of the system B+C just after A and B made contact. This velocity of the center of mass will not change in the subsequent motion. Express your answer through v_1 and m. **1 pt.**

In the frame fixed to the ground the instantaneous velocity of B is v_1 and that of C is 0, so

$$v_{\text{CM}} = \frac{mv_1 + m \cdot 0}{m+m} = \frac{v_1}{2}. \tag{1.1.8}$$

1 pt.

ii. What are the instantaneous velocities v_B^{CM} and v_C^{CM} of the bodies B and C in the center of mass reference frame at the moment when A and B make contact? Express your answer through v_1 and m. **1 pt.**

Since the velocity in the reference frame fixed to the ground of B is v_1 and that of C is 0 the velocities in the reference frame fixed to their center of mass have to be

$$v_B^{\text{CM}} = v_1 - v_{\text{CM}} = \frac{v_1}{2}, \tag{1.1.9}$$

0.5 pt.

$$v_C^{\text{CM}} = 0 - v_{\text{CM}} = -\frac{v_1}{2}. \tag{1.1.10}$$

0.5 pt.

iii. At the moment of contact between A and B, the spring is in its equilibrium position. What is the energy E^{CM} and amplitude A (see figure 1.1.11) of the ensuing oscillations in the center of mass system? Express your answer in terms of v_1, m and k.

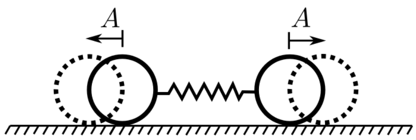


Figure 1.1.11: Definition of the amplitude of the oscillations.

3 pt.

After the collision the system B+C is isolated from A and thus its total energy has to be conserved. In the center of mass frame all the energy is in the oscillatory motion of the bodies B and C. This is because by being in the center of mass frame we have gotten rid of the translational motion. 1 pt.

Since we know the velocities of B and C at the moment after the collision, this gives us the energy as

$$E^{\mathrm{CM}} = \frac{m}{2}\left(\left(\frac{v_1}{2}\right)^2 + \left(-\frac{v_1}{2}\right)^2\right) = \frac{mv_1^2}{4}. \tag{1.1.12}$$

0.5 pt.

The fact that at the moment of collision the spring is at equilibrium means that at this moment the full energy is kinetic. At the point in the oscillation cycle where the bodies reach the amplitude the full energy will be stored in the spring. With the given definition of amplitude we have

$$\frac{mv_1^2}{4} = \frac{1}{2}k(2A)^2. \tag{1.1.13}$$

1 pt.

Solving for A gives

$$A = \sqrt{\frac{mv_1^2}{8k}}. \tag{1.1.14}$$

0.5 pt.

Part C. Lost energy *6 pt.*

Going back to the reference frame that is fixed to the ground, we take a look at the collision between the body A and the composite system B+C with mass $2m$ and velocity v_{CM}. The total kinetic energy is the sum of the kinetic energy of body A and the kinetic energy of the composite system.

i. What is the difference Q in the total kinetic energy before and after the collision? Express your answer in terms of V, m and M. **2.5 pt**

From the perspective of the collision of A and B+C in the reference frame fixed to the ground initially we have the energy

$$E_{\mathrm{i}} = \frac{1}{2}MV^2, \tag{1.1.15}$$

0.5 pt.

while after the collision we have the energy

$$E_\mathrm{f} = \frac{1}{2}MV'^2 + \frac{1}{2}(2m)v_\mathrm{CM}^2 + Q, \tag{1.1.16}$$

where Q denotes the energy lost to the internal motion of the B+C composite system. 0.5 pt.

Including Q, as we did above, means that $E_\mathrm{f} - E_\mathrm{i} = 0$, from which we can find a formula for Q as

$$Q = \frac{MV^2}{2} - \frac{MV'^2}{2} - mv_\mathrm{CM}^2. \tag{1.1.17}$$

0.5 pt.

Using the formulas we found in previous parts

$$V' = \frac{M-m}{M+m}V, \qquad v_\mathrm{CM} = \frac{v_1}{2} = \frac{MV}{M+m}, \tag{1.1.18}$$

we get

$$Q = \frac{MV^2}{2}\left(1 - \frac{2mM}{(M+m)^2} - \left(\frac{M-m}{M+m}\right)^2\right) = \frac{MV^2}{2}\frac{2Mm}{(M+m)^2}. \tag{1.1.19}$$

1 pt.

ii. Prove that the “lost” energy Q is equal to the energy of the internal motion of the system B+C in the center of mass reference frame of B+C. **1 pt.**

We express E^CM in terms of V, M and m and get

$$E^\mathrm{CM} = \frac{M^2mV^2}{(M+m)^2} = Q. \tag{1.1.20}$$

1 pt.

iii. We define q as the ratio between the “lost” energy Q and the initial kinetic energy of A. Express q in terms of the ratio $\alpha = \frac{M}{m}$. **1 pt.**

From the other subtasks we have

$$q = \frac{Q}{E_\mathrm{i}} = \frac{2Mm}{(M+m)^2}, \tag{1.1.21}$$

0.5 pt.

which we can express in terms of α

$$q = \frac{2\alpha}{(1+\alpha)^2}. \tag{1.1.22}$$

0.5 pt.

iv. For which value of α is the fraction q the highest? **1.5 pt**

We notice that the ”lost” energy Q is equal to the translational energy of the composite system. Consequently, the largest loss in energy is observed for the largest possible energy transfer with respect to the initial kinetic energy, which is the case when all kinetic energy goes from A to B, i.e. M = m. 1.5 pt.

Alternatively we can calculate the derivative of q with respect to α:

$$\frac{\mathrm{d}q}{\mathrm{d}\alpha} = \frac{2}{(1+\alpha)^2} - \frac{4\alpha}{(1+\alpha)^3} = 0. \tag{1.1.23}$$

Solving for α we get

$$\alpha = 1. \tag{1.1.24}$$

Problem 1.2: Condensed water vapor *16 pt.*

We consider two containers of $17.5\,\mathrm{L}$ **filled with** $n_0 = 1\,\mathrm{mol}$ **of water vapor at a pressure of** $2\,\mathrm{bar}$**. We now want to find out how we can condense the water vapor to liquid water by cooling it. To perform the calculations, we assume in the following that water vapor can be described as an ideal gas with three degrees of freedom. In addition, the following measurements for water shall be given**

- **Molar mass of water:** $18\,\mathrm{g}\cdot\mathrm{mol}^{-1}$
- **Density of water:** $1000\,\mathrm{kg}\cdot\mathrm{m}^{-3}$

For simplicity, assume that the density of water is independent of the pressure and temperature in the range that is relevant for this task. You are also given a separate document where a section of the p-T **phase diagram for water is plotted.**

Part A. Cooling *8 pt.*

i. What is the temperature of the water vapor in the containers? Draw this initial state in the attached p-T **diagram.** **2 pt.**

From the ideal gas law we have

$$T = \frac{pV}{nR}.$$

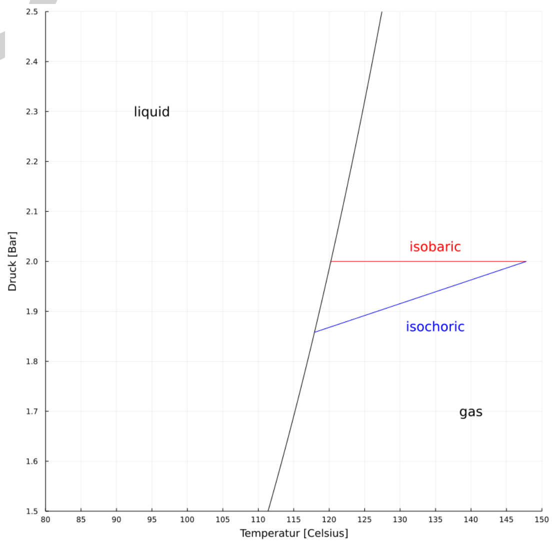
1 pt.

The numerical value is $T = 147.8\,^\circ\mathrm{C}$. 0.5 pt.

The point is correctly drawn in the diagram. 0.5 pt.

We now want to make the water vapor in the two containers condense. To do this, we cool one container isochorically and the other isobarically until they just begin to condense.

ii. Draw these two processes in the attached p-T **diagram.** **3 pt.**



For the isobaric process, we have a horizontal line until it crosses the saturation curve. 1 pt.

The isochoric process is a straight line with slope $\frac{nR}{V}$. 1 pt.

The isochoric process is correctly drawn in the p-T diagram. It is easiest to calculate another point at T_1 with

$$p_1 = T_1 \frac{p_0}{T_0},$$

where T_0, p_0 are the initial points. Then the data points (T_0, p_0) and (T_1, p_1) can be connected. 1 pt.

iii. At what pressure and temperature does the water vapor in the containers begin to condense? 1 pt.

We take the intersection of the processes with the saturation curve and read off the corresponding temperature and pressure.

We get $T_{t,V} = 118\,°\mathrm{C}$, $p_{t,V} = 1.86\,\mathrm{bar}$ for the isochoric process. $T_{t,p}$ has to be within $\pm 1\,°\mathrm{C}$ of the correct value and $p_{t,V}$ has to be within $\pm 0.03\,\mathrm{bar}$. 0.5 pt.

And $T_{t,p} = 120\,°\mathrm{C}$, $p_{t,p} = 2\,\mathrm{bar}$ for the isobaric process. $T_{t,p}$ has to be within $\pm 1\,°\mathrm{C}$ of the correct value. 0.5 pt.

iv. Calculate how much heat was extracted from the water vapor in both processes. In which process less heat was extracted? 2 pt.

For the isochoric process we have

$$Q_V = C_V \Delta T = \frac{3}{2} nR(T_0 - T_{t,V}).$$

0.5 pt.

Numerical application gives $Q_V = 373\,\mathrm{J}$. 0.5 pt.

For the isobaric process we have

$$Q_p = C_p \Delta T = \frac{5}{2} nR(T_0 - T_{t,p}).$$

0.5 pt.

Numerical application gives $Q_p = 573\,\mathrm{J}$. 0.5 pt.

Part B. Phase transition *6 pt.*

In the following, we will only consider the container which we have cooled isobarically so far. We continue to cool this container isobarically in order to let the water vapor condense. This means that the pressure p_t remains constant and therefore also the temperature T_t is fixed by the vapor pressure curve during the phase transition. During the phase transition, the liquid and gaseous phases coexist. We denote the respective volumes by V_{li} and V_{gas}. We say that the phase transition is complete when all vapor has condensed to water.

i. Let n_0 be the number of gas molecules before the phase transition begins and n_{gas} be the number of gas molecules during the phase transition. What is the number of molecules in the liquid phase? 0.5 pt

We cannot exchange any particles with the environment therefore the amount of particles is conserved. So we have $n_0 - n_{\text{gas}}$ mole molecules in the liquid phase. 0.5 pt.

ii. What are the volumes V_{li} and V_{gas} at the beginning of the phase transition? **1.5 pt**

There is no water at the beginning,

$$V_{\text{li},i} = 0.$$

0.5 pt.

From the ideal gas law,

$$V_{\text{gas},i} = \frac{nRT_t}{p_t}.$$

0.5 pt.

The numerical result is

$$V_{\text{gas},i} = 16.4\,\text{L}.$$

0.5 pt.

iii. What are the volumes V_{li} and V_{gas} after the phase transition is complete? **1.5 pt**

We have

$$V_{\text{gas},f} = 0.$$

0.5 pt.

We have n_0 mole particles in water. Therefore

$$V_{\text{li},f} = \frac{n_0 M_{water}}{\rho}.$$

where ρ is the density. 0.5 pt.

The numerical result is

$$V_{\text{li},f} = 18\,\text{mL}.$$

0.5 pt.

iv. Calculate the mechanical work exerted on the gas during the phase transition. **1 pt.**

Since the phase transition is isobaric the mechanical work is

$$W = p\Delta V = p(V_{\text{gas},i} - V_{\text{li},f}) \approx pV_{\text{gas},i}.$$

0.5 pt.

The numerical value is

$$W = 3271\,\text{J}.$$

0.5 pt.

We now want to find out in more detail what happens during the phase transition.

v. Find a formula for the volume of the system during the phase transition as a function of the instantaneous number of water vapor particles $n_{\text{gas}} < n_0$, the number of water vapor particles n_0 at the beginning of the phase transition, the pressure p_t, and the temperature T_t. **1.5 pt**

The volume of the gaseous part is by ideal gas law

$$V_{\text{gas},t} = \frac{n_{\text{gas}} R T_t}{p_t}.$$

0.5 pt.

The volume of the liquid part is

$$V_{\text{li},t} = \frac{(n_0 - n_{\text{gas}})M}{\rho}.$$

0.5 pt.

We therefore get

$$V_t = \frac{T_t}{p_t}\left(n_{\text{gas}} R + \frac{p_t}{T_t \rho} M(n_0 - n_{\text{gas}})\right).$$

0.5 pt.

Part C. Isochoric phase transition *2 pt.*

i. Describe qualitatively what happens if we cool isochorically instead of isobarically during the phase transition. 1 pt.

The ratio between vapor and water will decrease by extracting more heat, but since we have fixed volume there will always be some gas left filling out the container. This is possible, because the pressure decreases as well and therefore the gaseous phase is less dense. 1 pt.

ii. Draw this phase transition process in the p-T diagram. 1 pt.

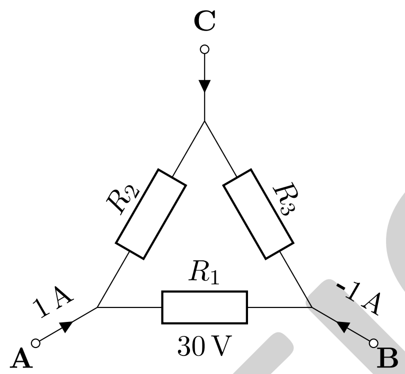
Since we have coexistence of water and vapor we have to be on the water vapor curve. This means the process will follow the water vapor curve. 1 pt.

Problem 1.3: Electric amplifier *16 pt.*

In many electrical circuits, signals have to be amplified. Be it the weak signal of an antenna or to turn the music louder. In this task we want to look at how one can set the gain of certain electrical amplifiers to a given value.

Part A. Warm-up problem *2 pt.*

To prepare ourselves for the following tasks, let us briefly review the two Kirchhoff rules. For this purpose we analyze the following circuit with three contacts A, B and C:



Based on measurements, we know that a current of 1 A flows into the circuit at contact A, that a current of 1 A flows out at contact B and that the voltage across resistor R_1 is 30 V.

i. If the resistance R_3 is twice as large as R_2, what is the voltage across R_2? **2 pt.**

We can look at the whole circuit as one knot. Therefore we can deduce that there is no current at point C, from the first Kirchhoff rule. This means the current through R_2 and R_3 is the same. 0.5 pt.

From the second Kirchhoff rule we know that the combined voltage over R_2 and R_3 is equal to the voltage over R_1. 0.5 pt.

Since there is no current flowing out in point C, R_2 and R_3 split the voltage with a ratio one to two. 0.5 pt.

Therefore the voltage over R_2 is 10 V. 0.5 pt.

Part B. Adjust the gain *8 pt.*

In this task we consider an inverting amplifier (see Fig. 1.3.1) and make the following assumptions:

- **The input voltage and output voltage have different signs (hence the name "inverting amplifier").**
- **The input voltage U_{in} is amplified by a factor $k_0 = -10^6$.**
- **The amplifier is connected to a power supply with 12 V and −12 V (not drawn) and can produce output voltages only in this range.**
- **The input resistance (internal resistance between input and ground) into the amplifier is very large with $R_i = 10\,\mathrm{M\Omega}$.**

i. What is the maximum voltage range ΔU that can be applied at the input for the amplifier to still work properly?

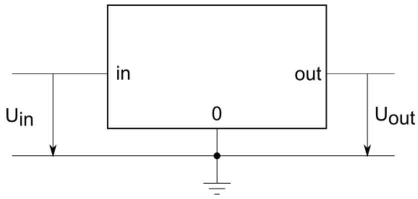


Figure 1.3.1: The rectangle symbolizes the amplifier, the output voltage U_{out} is the input voltage U_{in} amplified by k_0.

1 pt.

The maximal output range is $24\,\text{V}$. Taking the magnification k into account, the maximal input range is $\Delta U = 24\,\text{V}/k_0 = 24\,\mu\text{V}$. 1 pt.

If only taking the output range as $12\,\text{V}$*, only reward half the points.*

ii. If a voltage of $U_{\text{in}} = 1\,\mu\text{V}$ is applied to the input, how much current flows through the input (in)? **1 pt.**

Since the input impedance is $R_i = 10\,\text{M}\Omega$, the current is $I = \frac{U}{R} = 10\,\text{fA} = 1 \times 10^{-13}\,\text{A}$. 1 pt.

iii. By adding two resistors $R_1, R_2 \ll R_i$, the amplification can be set to any value $k < k_0$, see Fig. 1.3.2. What is the relationship (to a good approximation) between the currents I_1 and I_2?

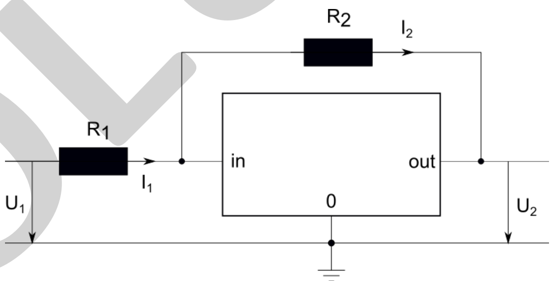


Figure 1.3.2: The amplifier with the two additional resistors R_1 and R_2. In addition, the applied voltage U_1 and the current I_1 through R_1, as well as analogously the voltage U_2 and current I_2 are drawn.

2 pt.

Since the resistance $R_1, R_2 \ll R_i$, we can neglect R_i for current considerations, i.e. no current flows into the input. 0.5 pt.

Then applying Kirchoff's current law, the current I_1 flowing into the amplifier has to be the same as the one flowing away, i.e. I_2. 1 pt.

Therefore $I_1 = I_2$ (give this point also if it is obvious from the explanation). 0.5 pt.

Applying Kirchhoff as a sum, one might also find that $I_1 = -I_2$, which is considered here as equivalent.

iv. Suppose we want to achieve a gain $k \ll k_0$. What is the ratio between U_1 and U_2, resp. what is the resulting gain as a function of R_1 and R_2? **1.5 pt**

Making use of the currents being equal ($I_1 = I_2 = I$), we get $\frac{U_2}{U_1} = -\frac{IR_2}{IR_1} = -\frac{R_2}{R_1}$. 1 pt.

The amplification is therefore $k = -\frac{R_2}{R_1}$. 0.5 pt.

Note the minus sign: On one hand this is due to the inverting amplification. On the other hand it it due to the definition of I_1 and I_2, i.e. to apply Kirchhofs law.

v. What voltage U_{in} is applied to the amplifier? **1 pt.**

There are two approaches, both are equivalent:

From a controller point of view, one can imply that the amplifier sets the output voltage given by U_1, R_1 and R_2 always such that the voltage at the input U_{in} is zero. Alternatively from the above calculation (with right signs, i.e. $\text{sgn}(U_1) = -\text{sgn}(U_2)$ one sees that the voltage between the two resistors (i.e. the input energy of the amplifier) is always zero. 1 pt.

vi. Suppose you want to amplify a weak signal with an input voltage $U_1 = 20\,\text{mV}$ by a factor $k = 100$ so that the signal has a maximum current of $10\,\mu\text{A}$. What values R_1 and R_2 do you choose? Does the approximation $R_1, R_2 \ll R_i$ still apply? **1.5 pt**

In order not to exceed the input current of 10µA, we have to chose $R_1 > \frac{U_1}{I_1} = 2000\,\Omega$. 0.5 pt.

On the other hand the output resistor is given by $R_2 = kR_1$. 0.5 pt.

Since $R_1 \ll R_2 \ll R_i$, the approximation is still valid. 0.5 pt.

Part C. Slew rate *6 pt.*

We go back to the amplifier without additional resistors, see Fig. 1.3.1. Usually amplifiers cannot boost the input voltage arbitrarily fast, but need a certain time to react. The slew rate indicates the maximum rate at which the voltage at the output (out) of the amplifier can rise.

i. Assume that the slew rate is $10\,\text{V} \cdot \mu\text{s}^{-1}$. What is approximately the maximum frequency that can still be amplified? Note: Assume that the amplitude of the output signal is about $1\,\text{V}$ and the input signal is sinusoidal. **2 pt.**

The maximal slope of a sinusoidal oscillation with angular frequency ω and amplitude $1\,\text{V}$ is $1\omega\text{V}$. 1 pt.

Equating this with the given raising time leads to the maximal frequency: $\omega = 10\,\mu\text{s}^{-1} = 1 \times 10^{-5}\,\text{s}^{-1}$. 0.5 pt.

Therefore the frequency is of about $f = 1.6\,\text{MHz}$. 0.5 pt.

Note that in the end the order of magnitude is important. So if there is a good reasoning for a different frequency close to $f = 1.6\,\text{MHz}$, it is also fine.

ii. Suppose we immediately change the voltage at the input (in) from $0\,\text{V}$ to $10\,\mu\text{V}$ at $t = 0$. How does the output voltage look like (use numerical quantities given in this task)? Make a graph. **4 pt.**

The maximal output voltage is $10\,\text{V}$. So for big times, it should converge to there. 0.5 pt.

Before $t = 0$, the output voltage is zero. 0.5 pt.

The raise of the voltage has to happen within about $1\,\mu\text{s}$. Whether the change is given by an exponential or a straight line does not matter. 1 pt.

Labelling of x-axis 0.5 pt.

Labelling of y-axis 0.5 pt.

X-axis scaled 0.5 pt.

Y-axis scaled 0.5 pt.

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Physics Olympiad

Second Round

online, 19 January 2021

Part 2	**:**	**20 MC questions**
Duration	:	60 minutes
Total	:	20 points (20×1)
Authorized material	:	Calculator without database
		Writing and drawing material
		One A4 double-sided handwritten page of notes

Good luck!

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Natural constants

Caesium hyperfine frequency	$\Delta\nu_{\mathrm{Cs}}$	9.192 631 770	$\times 10^{9}$	s^{-1}
Speed of light in vacuum	c	2.997 924 58	$\times 10^{8}$	$\mathrm{m \cdot s^{-1}}$
Planck constant	h	6.626 070 15	$\times 10^{-34}$	$\mathrm{kg \cdot m^{2} \cdot s^{-1}}$
Elementary charge	e	1.602 176 634	$\times 10^{-19}$	$\mathrm{A \cdot s}$
Boltzmann constant	k_{B}	1.380 649	$\times 10^{-23}$	$\mathrm{K^{-1} \cdot kg \cdot m^{2} \cdot s^{-2}}$
Avogadro constant	N_{A}	6.022 140 76	$\times 10^{23}$	$\mathrm{mol^{-1}}$
Luminous efficacy of radiation	K_{cd}	6.83	$\times 10^{2}$	$\mathrm{cd \cdot kg^{-1} \cdot m^{-2} \cdot s^{3} \cdot sr}$
Magnetic constant	μ_0	1.256 637 062 12(19)	$\times 10^{-7}$	$\mathrm{A^{-2} \cdot kg \cdot m \cdot s^{-2}}$
Electric constant	ε_0	8.854 187 812 8(13)	$\times 10^{-12}$	$\mathrm{A^{2} \cdot kg^{-1} \cdot m^{-3} \cdot s^{4}}$
Gas constant	R	8.314 462 618...		$\mathrm{K^{-1} \cdot kg \cdot m^{2} \cdot mol^{-1} \cdot s^{-2}}$
Stefan-Boltzmann constant	σ	5.670 374 419...	$\times 10^{-8}$	$\mathrm{K^{-4} \cdot kg \cdot s^{-3}}$
Gravitational constant	G	6.674 30(15)	$\times 10^{-11}$	$\mathrm{kg^{-1} \cdot m^{3} \cdot s^{-2}}$
Electron mass	m_{e}	9.109 383 701 5(28)	$\times 10^{-31}$	kg
Neutron mass	m_{n}	1.674 927 498 04(95)	$\times 10^{-27}$	kg
Proton mass	m_{p}	1.672 621 923 69(51)	$\times 10^{-27}$	kg
Standard acceleration of gravity	g_{n}	9.806 65		$\mathrm{m \cdot s^{-2}}$

Multiple Choice: answer sheet

Duration: 60 minutes
Marks: 20 points (1 point for each correct answer)

Indicate your answers in the corresponding boxes on this page.
Label the sheets with your name and the number of the problem. Furthermore, number your sheets.

- Multiple-Choice **(MC)** questions have several statements, of which **exactly one** is correct. If you mark exactly the right answer on the answer sheet, you get one point, otherwise zero.

Last name:	**First name:**	**Total:**

	A)	B)	C)	D)	E)	F)
Question 2.1	☐	☐	☐	☐		
Question 2.2	☐	☐	☐	☐	☐	
Question 2.3	☐	☐	☐	☐	☐	
Question 2.4	☐	☐	☐	☐		
Question 2.5	☐	☐	☐	☐		
Question 2.6	☐	☐	☐	☐	☐	
Question 2.7	☐	☐	☐	☐	☐	
Question 2.8	☐	☐	☐	☐	☐	
Question 2.9	☐	☐	☐	☐	☐	
Question 2.10	☐	☐	☐	☐		
Question 2.11	☐	☐	☐	☐	☐	
Question 2.12	☐	☐	☐			
Question 2.13	☐	☐	☐	☐	☐	
Question 2.14	☐	☐	☐	☐	☐	
Question 2.15	☐	☐	☐	☐	☐	
Question 2.16	☐	☐	☐	☐		
Question 2.17	☐	☐	☐	☐		
Question 2.18	☐	☐	☐	☐	☐	
Question 2.19	☐	☐	☐	☐	☐	☐
Question 2.20	☐	☐	☐	☐		

Multiple Choice: questions

Question 2.1 (MC)

How many eggs are annually consumed by humanity?

A) 7.67×10^7 t B) 7.67×10^8 t
C) 7.67×10^9 t D) 7.67×10^{10} t

Question 2.2 (MC)

What is the mean maximal power output of a cyclist (over a short time)?

A) 10 W B) 100 W C) 1000 W
D) 10 000 W E) 100 000 W

Question 2.3 (MC)

The temperature T of a black-hole (the so-called Hawking radiation temperature) can essentially be written as a function of its mass M and the fundamental constants h (Planck constant), c (speed of light), k_B (Boltzmann constant) and G (gravitational constant). If we forget about any numerical constants, T is therefore proportional to:

A) $\frac{hcG}{k_B M}$ B) $\frac{hc^6}{G^2 M^2}$ C) $\frac{hc^2}{G^2 k_B}$ D) $\frac{k_B c^2}{GMh}$ E) $\frac{hc^3}{Gk_B M}$

Question 2.4 (MC)

Sabrine doesn't like ironing clothes at all. That's why she gets Maurice to help her. Knowing that it takes her 1h30 and Maurice 3h to iron the laundry alone, how much time do they need together to fulfill the task?

A) 50 min B) 60 min C) 70 min D) 80 min

Question 2.5 (MC)

An ice floe (density $920\,\mathrm{kg \cdot m^{-3}}$) floating in water (density $1000\,\mathrm{kg \cdot m^{-3}}$) has the shape of a vertical cylinder. In one day it melts so much that the part sticking out of the water is 10 cm shorter (but still has the shape of a cylinder). How much has it melted below the surface of the water?

A) 10 cm B) 85 cm C) 115 cm D) 230 cm

Question 2.6 (MC)

On which quantities does the acceleration due to gravity (g) depend?

A) On none, it is a fundamental constant.
B) On the distance between the Sun and the Earth.
C) On the mass of the Sun.
D) On the altitude.
E) On the atmospheric pressure.

Question 2.7 (MC)

A dam is to be built to protect against a flood of Lake Biel (area $39\,\mathrm{km^2}$, perimeter 45 km). What pressure must the dam withstand to contain an overflow of 10 cm?

A) 9.8 hPa B) 8.5 MPa C) 61 MPa
D) 440 MPa E) 3.8 TPa

Question 2.8 (MC)

At a leisurely lunch you hear from the opposite table that a blue whale can grow up to 33 m long and weigh 200 t. What is the diameter of the whale? For simplicity's sake, assume that the whale is made of water and is shaped like a cylinder.

A) 0.75 m B) 1.5 m C) 3 m D) 5 m E) 10 m

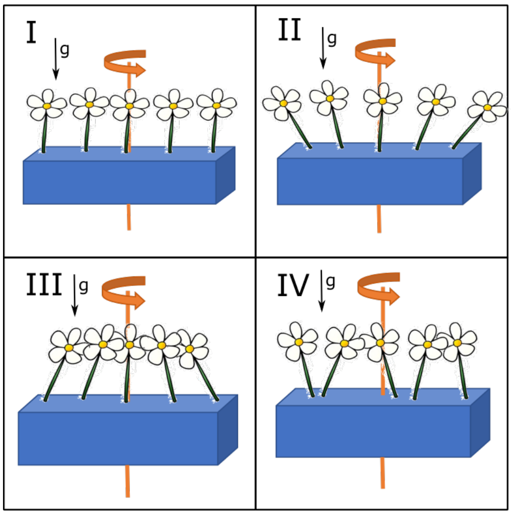
Question 2.9 (MC)

The heart of our blue whale pumps up to 5000 L of blood a minute with only about 5 heartbeats. The aorta has a diameter of 20 cm. How great must the flow velocity of the blood be for this (assume a constant flow velocity over the entire cross-section)?

A) $50\,\mathrm{km \cdot h^{-1}}$ B) $10\,\mathrm{km \cdot h^{-1}}$ C) $5\,\mathrm{km \cdot h^{-1}}$
D) $1\,\mathrm{km \cdot h^{-1}}$ E) $0.5\,\mathrm{km \cdot h^{-1}}$

Question 2.10 (MC)

As you have certainly already observed, flowers normally grow vertically upwards, regardless of whether they grow on a surface or on a slope. Physically, this can be formulated in such a way that they always grow opposite to the time-averaged net force acting on them. We now place a flower pot on a rotating plate. What is the growth form of the new flowers (they were still very small when they were placed on the rotating plate)?



A) I B) II C) III D) IV

Question 2.11 (MC)

Barbara has a lens with a focal length of 30 cm. She wants to start a fire. How far away does she need to hold the lens from the wood?

A) As close as possible.

B) 15 cm

C) 30 cm

D) 60 cm

E) As far away as possible.

Question 2.12 (MC)

Astronaut Gabriel took a Rivella with him to the Moon for refreshment. During the journey, however, the Rivella was severely shaken. Gabriel was told that on Earth the bottles have a burst pressure of about 5 bar. The carbonic acid released by the shaking increases the partial pressure of the CO_2 in the bottle by 3.5 bar. Can Gabriel take the Rivella out of the spaceship through the airlock and take a selfie with it without getting his spacesuit dirty (from the outside)?

A) No, the bottle will burst.

B) Yes, no problem.

C) No, the bottle will implode.

Question 2.13 (MC)

A distracted physicist has ordered 2 dm^3 bread from the bakery. The baker knows you are familiar with thermodynamics and asks you to help him bake the bread.
He remembers that his dough has doubled in volume from rising and had an initial volume of 0.8 L. At what temperature must the bread be baked for it to reach the desired volume as closely as possible? Assume that the bread reaches the temperature of the oven immediately and that the pressure in the bread is always 1 bar. Assume all the original gas remains in the bread. Ignore the thermal expansion of solids.

A) 125 °C B) 175 °C C) 225 °C

D) 275 °C E) 325 °C

Question 2.14 (MC)

There is 1 kg of boiling water in an open container. We add a piece of lead with a mass of 0.5 kg and an initial temperature of 250 °C into the water. What can we say about the temperature T_w of the water and T_l of the lead right after we added the piece of lead?

A) T_w decreases and T_l remains constant.

B) T_w increases and T_l remains constant.

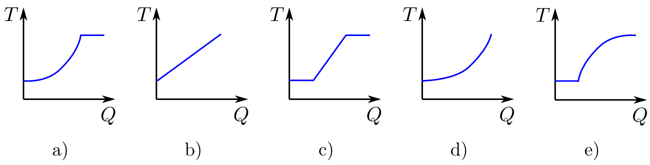
C) T_w increases and T_l decreases.

D) T_w remains constant and T_l decreases.

E) T_w remains constant and T_l increases.

Question 2.15 (MC)

Which graph best describes the relationship between the mean kinetic energy E_k of an ideal gas molecule and its absolute temperature T?



A) a B) b C) c D) d E) e

Question 2.16 (MC)

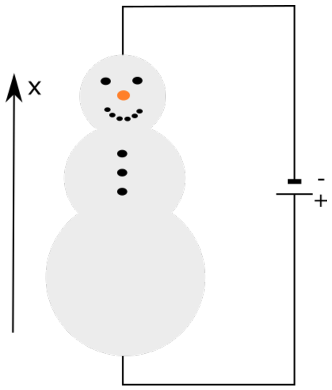
Mathilde is building a small electric circuit to amplify a signal. She needs a resistor of exactly $72\,\mathrm{m\Omega}$. Since she didn't find any, she decided to create it herself with some copper wire with diameter $0.5\,\mathrm{mm}$. Knowing that copper's conductance is $\sigma = 5.95 \times 10^{7}\,\Omega^{-1} \cdot \mathrm{m}^{-1}$, how much length of wire does she need?

A) $5.90 \times 10^{-1}\,\mathrm{m}$ B) $8.40 \times 10^{-1}\,\mathrm{m}$

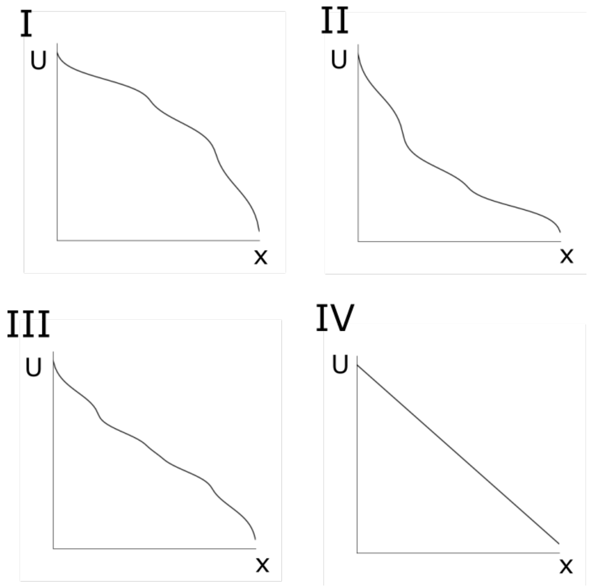
C) $1.19\,\mathrm{m}$ D) $1.69\,\mathrm{m}$

Question 2.17 (MC)

You have a resistance shaped like a snowman.



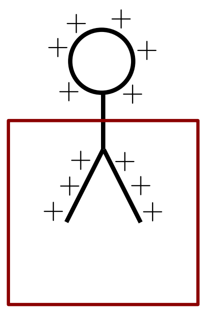
What will the voltage drop look like as function of distance?



A) I B) II C) III D) IV

Question 2.18 (MC)

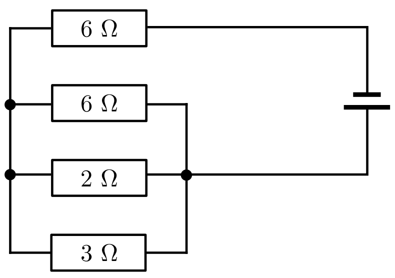
A leaf electroscope is a device used to detect the presence of electric charges. It consists of a metallic knob and two small metallic leaves inside a vacuum chamber (in red) connected to the knob by a metallic rod. The leaves can move freely. The picture shows a *positively* charged electroscope. Now let's assume that we approach a *positively* charged glass rod close to the knob of this electroscope, but without touching it. What can we say about the knob charge and the distance between the leaves?



A) The charge on the knob decreases and the distance between the leaves decreases.
B) The charge on the knob decreases but the distance between the leaves increases.
C) The charge on the knob increases and the distance between the leaves increases.
D) The charge on the knob increases but the distance between the leaves decreases.
E) Nothing changes.

Question 2.19 (MC)

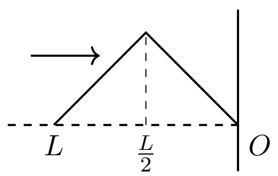
In the following circuit, we know that a current of 3 A is flowing through the 2 Ω resistor. What is the voltage delivered by the voltage source?



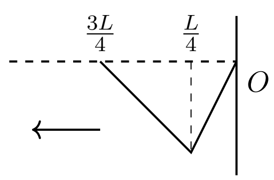
A) $U = 12\,\text{V}$ B) $U = 21\,\text{V}$ C) $U = 24\,\text{V}$
D) $U = 30\,\text{V}$ E) $U = 42\,\text{V}$ F) $U = 51\,\text{V}$

Question 2.20 (MC)

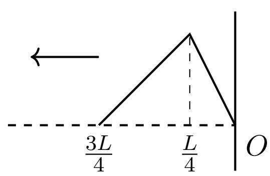
A triangular shaped pulse of length L is completely reflected at the fixed end of the string on which it travels. What will be the shape of the pulse after a length $\frac{3L}{4}$ of the pulse has been reflected?



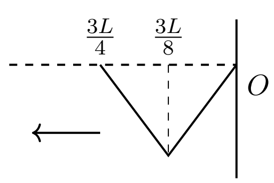
A)



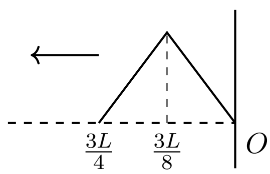
B)



C)



D)



Multiple Choice: solutions

	A)	B)	C)	D)	E)	F)
Question 2.1	■	□	□	□		
Question 2.2	□	□	■	□	□	
Question 2.3	□	□	□	□	■	
Question 2.4	□	■	□	□		
Question 2.5	□	□	■	□		
Question 2.6	□	□	□	■	□	
Question 2.7	■	□	□	□	□	
Question 2.8	□	□	■	□	□	
Question 2.9	□	■	□	□	□	
Question 2.10	□	□	■	□		
Question 2.11	□	□	■	□	□	
Question 2.12	□	■	□			
Question 2.13	□	■	□	□	□	
Question 2.14	□	□	□	■	□	
Question 2.15	□	■	□	□	□	
Question 2.16	□	■	□	□		
Question 2.17	■	□	□	□		
Question 2.18	□	■	□	□	□	
Question 2.19	□	□	□	□	■	□
Question 2.20	■	□	□	□		